DTIC FILE COPY

(12)

AD-A181 327

NECK MUSCLE ENDURANCE AND FATIGUE AS A FUNCTION OF HELMET LOADING: THE DEFINITIVE MATHEMATICAL MODEL

Annual and Final Report

Chandler Allen Phillips, M.D., P.E.
Jerrold Scott Petrofsky, Ph.D.

September, 1984

Supported by:

U.S. Army Medical Research and Development Command
Fort Detrick, Frederick MD 21701-5012

Contract No. DAMD17-80-C-0089

Wright State University
Dayton, Ohio 45435

DOD Distribution Statement

Approved for public release; distribution unlimited.

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

87 6 070

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION: UNCLASSIFIED	1b. RESTRICTIVE MARKINGS: A181327
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT: Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION: Wright State University; 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION
6c. ADDRESS (City, State, and ZIP Code): Dayton, Ohio 45435	7b. ADDRESS (City, State, and ZIP Code)
8a. NAME OF FUNDING/SPONSORING ORGANIZATION: U.S. Army Medical Research & Development Command; 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER: DAMD17-80-C-0089
8c. ADDRESS (City, State, and ZIP Code): Fort Detrick, Frederick, Maryland 21701-5012	10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT ACCESSION NO.
62777A	3F1-62777A878	AG	150

11. TITLE (Include Security Classification)
Neck Muscle Endurance and Fatigue as a Function of Helmet Loading: The Definitive Mathematical Model

12. PERSONAL AUTHOR(S) Phillips, Chandler Allen and Petrofsky, Jerrold Scott

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
Annual and Final	FROM 80/6/1 TO 84/6/15	84/9	88

16. SUPPLEMENTARY NOTATION

17. COSATI CODES		
FIELD	GROUP	SUB-GROUP
05	05	
05	08	(see reverse)

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
Mathematical model, Computer simulation, Neck muscles, Helmet design, Fatigue

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

A series of experiments were conducted in which the neck muscles of volunteer subjects were dynamically and statically loaded by systematic variations of twenty-four headgear configurations consisting of eight different centers-of-gravity (CGs) times three different weights. Six subjects would rotate their heads laterally (from side-to-side) for 30 min with each of the headgear loading combinations. Immediately thereafter, the subject would position his head in an isometric head dynamometer and exert a sustained right lateral neck contraction or forward neck contraction at 70% of his maximum strength, during which endurance time (to fatigue) was recorded. The results indicate that the computer model makes reasonable predictions within the boundary conditions. Input data outside the boundary conditions is rejected. The assumption of insensitivity to vertical loading is demonstrated. The assumption of bilateral symmetric response was confirmed for the 1.45 kg and 2.27 kg helmet loads. However, this assumption was not confirmed for the 4.09 kg helmet load. It is concluded from the computer model that afterward, midline loading is the optimal CG location (i.e. maximal endurance) for heavier helmets in the 3-4 kg range.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT: ☐ UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION: UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL: Jane B. Idoine	22b. TELEPHONE (Include Area Code): 301/663-7325; 22c. OFFICE SYMBOL: SGRD-RMS

DD FORM 1473, 84 MAR — 83 APR edition may be used until exhausted. All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE

Block 17 (continued):

Field	Group
06	16

SUMMARY

A series of experiments have been conducted to supplement our current data bank of helmet loading configurations (then at fifteen different helmet weight and center-of-gravity combinations). The neck muscles were dynamically and statically loaded by systematic variation of nine additional headgear configurations consisting of three different combinations of centers-of-gravity (right-forward-low, left-lateral-low and right-aftward-low) and three different weights (1.45 kg., 2.27 kg. and 4.09 kg.). Six subjects would rotate their heads laterally (from side-to-side) for 30 minutes with each of the headgear loading combinations. Immediately thereafter, the subject would position his head in an isometric head dynamometer and exert a sustained right lateral neck contraction or forward neck contraction at 70% of his maximum strength, during which endurance time (to fatigue) was recorded, the EMG over the right sternocleidomastoid muscle, over the posterior trapezius/splenius muscles, and the systolic and diastolic blood pressure and heart rate were continuously recorded. Of the twenty-four resultant combinations of helmet weights and centers-of-gravity: (1) 18 combinations were used as the boundary conditions for an empirical mathematical model to predict both forward and lateral neck muscle endurance for any weight-C.G. combination within the boundary conditions; (2) 3 combinations were used to test the assumption of insensitivity to vertical loading; and (3) 3 combinations were used to test the assumption of bilateral symmetric response. Finally, a statistical analysis program was supplied to perform paired T-tests for comparison of different headgear loading configurations. The results indicate that the mathematical model makes reasonable predictions within the boundary conditions. Input data outside the boundary conditions is rejected. The assumption of insensitivity to vertical loading is demonstrated. The assumption of bilateral symmetric response was confirmed for the 1.45 kg. and 2.27 kg. helmet loads. However, this assumption was not confirmed for the 4.09 kg. helmet load. It is concluded that the computer model is valid for midline, vertical and lateral headgear loading within the boundary conditions specified. (This work was supported by U.S. Army contract DAMD17-80-C-0089.)

FOREWORD

Citations of organizations and trade names in this report do not constitute an official Department of the Army endorsement of approval of the products or services of these organizations.

For the protection of human subjects the investigator(s) have adhered to policies of applicable Federal Law 45CFR46.

The authors wish to gratefully acknowledge the support received in this work from personnel of the United States Army Aeromedical Research Lab at Fort Rucker, Alabama. We wish to acknowledge the help of Harry Heaton and Iris Davis in these experiments.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

This section will outline the objectives of the research and its significance to the U.S. Army. The previous work by the investigators will be reviewed.

OBJECTIVES

The following objectives were addressed during the course of this study:

A. Supplement our current data bank of helmet loading configurations (then at 15 different helmet weight and center-of-gravity combinations) with additional experiments (9 additional helmet weight and center-of-gravity combinations) in order to define the necessary boundary conditions for a realistic mathematical model;

B. Develop an appropriate empirical mathematical model (using the technique of piece-wise linear analysis) to predict both forward and lateral neck muscle endurance for any weight-C.G. configuration within the boundary conditions;

C. Deliver to the U.S. Army, the results of this research in "software" package form that is (1) conveniently formatted, (2) easily accessible, and (3) readily interpretable.

SIGNIFICANCE

Our results are directly applicable to several objectives of the U.S. Army Aeromedical Research Laboratory (USAARL). It will provide for systematic understanding of how different headgear designs load the cervical muscles, affect the fatigue end-point, and therefore the subject's ability to tolerate various headgear loading. Consequently, it will enable better (or optimal) helmet design which will minimize both muscle loading and fatigue in the various driving or flying environments, and therefore maximize operational endurance time.

Pilots are currently being asked to wear and use additional headgear. For example, night vision goggles are now being worn in combination with a helmet. An objective evaluation of the cumulative effects of various helmet weights and center-of-gravity combinations on neck muscle tension and fatigue is now needed to establish the optimal "trade off" between headgear requirements and physiological capabilities.

Finally, the U.S. Army continues to design and evaluate new helmets for crew members. Impact protection, noise protection and visual protection (among other parameters) are all capable of objective quantitative evaluation with respect to helmet design. Successful application of this experimental project will yield a system of equations that will allow the designer to input important helmet design parameters (i.e., weight and center-of-gravity), and the equations will output isometric endurance time for the neck muscles in the forward and lateral contraction mode.

BACKGROUND

In our laboratory, support from Army contract DAMD17-80-C-0089, began in June, 1980. The Biomechanics Laboratory at Wright State University has subsequently developed a unique capability to evaluate neck muscle endurance and fatigue as a function of helmet loading. In pursuing our studies for the Army, two unique pieces of equipment were developed. First, an isometric helmet dynamometer was designed. This allows us to quantitate the

strength of neck muscle contractions in the forward, backward and both lateral modes (Phillips and Petrofsky 1981b, 1981c). Furthermore, it allows subjects to hold a "target" tension less than their maximal contraction tension while simultaneously measuring (1) endurance time, (2) neck muscle EMG, and (3) cardiovascular parameters. Second, a variable weight and variable C.G. helmet simulator was developed. This helmet simulator has been calibrated for five weights (1.4 to 4.1 kgs.) and twenty-one center-of-gravity locations (in the X, Y and Z plane). Furthermore, the helmet simulator can be recalibrated for any desired weight and C.G. combination within its design limits. The helmet simulator was validated during the first year of our army contract (Phillips and Petrofsky 1982b), and during our second year (commencing June, 1981), allowed us to observe the effects of fifteen different weight and C.G. combinations on neck muscle endurance and fatigue.

Between June, 1980 and May, 1981 a number of important parameters were evaluated. For the first time, our group determined the basic strength-endurance curves for neck muscle in forward, right-lateral, and backward contraction modes and compared them to the hand-grip muscles in the same subjects (Phillips and Petrofsky 1981a; Petrofsky and Phillips 1982). The research program then investigated the effects of no helmet (CONTROL), a standard SPH-4 helmet (HELMET), and the above helmet combined with Night-Vision-Goggles (NVG) on the cardiovascular responses (Phillips and Petrofsky 1982c), the neck muscle isometric endurance time and EMG response (Phillips and Petrofsky 1982a, 1983a). Finally, the variable center-of-gravity and variable weight helmet simulator (VCGW) was validated against the SPH-4 helmet and Night-Vision-Goggle combination (H/NVG) with respect to neck muscle endurance time and EMG response (Phillips and Petrofsky 1982b). The final contract report has not yet been released for general distribution pending some technical and format revisions (Phillips and Petrofsky 1981d).

Between June 1981 and May 1982, the isometric endurance time, EMG response, cardiovascular response (Phillips and Petrofsky 1983c, 1984b) and strength-recovery response were evaluated as a function of fifteen different helmet stimulator configurations. These represent a combination of three different helmet weights (i.e., 1.4, 2.3, 4.1 kg.) and five different centers-of-gravity (i.e., center-low [CL], center-high [CH], forward-low [FL], rearward-low [RL] and right-lateral-low [RLL], as shown graphically in Figure 1. After analysis of these results, it was concluded that there were optimal C.G. locations for 1.4 and 2.3 kg. (forward-low and right-lateral low) and for 4.1 kg. (aftward-low) (Phillips and Petrofsky 1983b). The final contract report for this period was submitted in November, 1982, and is still undergoing technical and editorial review by the U.S. Army (Phillips and Petrofsky 1982d).

It can be appreciated that any helmet system (whether current or projected) will have its own unique weight and C.G. location which will rarely fit one of the 15 combinations studied during the second year's work.

Furthermore, it would be highly impractical to test the large number of configurations required so that any present or projected helmet system could be closely approximated.

Referring again to Fig. 1, a conventional three-dimensional reference system is shown and those configurations noted which were determined at the conclusion of second-year study (O) and during the first half of the present study (●), see Methods and Materials section. The arrow notation refers

to positive movements about each axis, and numbers beside each C.G. configuration are weights (in pounds).

The additional data points (●) provided the additional boundary conditions necessary to apply piece-wise linear analysis. The resultant equations are capable of predicting the useful operational endurance time as a function of any helmet weight and center-of-gravity combination within the boundary conditions (Phillips and Petrofsky 1984a, 1986).

METHODS AND MATERIALS

This section describes how the study was conducted, including use of subjects. This section also includes a brief description of materials and apparatus used in the study. The fifteen headgear combinations reported for the second year study are combined with the nine headgear combinations evaluated in this final year study (a total of twenty-four headgear combinations). This was done since the objective of the final year study was a mathematical model which required all twenty-four headgear configurations in order to derive the model.

SUBJECTS

Six subjects were used in these experiments. The subjects were male volunteer university students whose ages, heights, neck sizes, and weights are listed in Table 1. All subjects were informed of all experimental procedures and were medically examined including a thorough history and a complete physical exam. All procedures were fully approved by the committee on human experimentation.

TRAINING

All subjects were first trained to produce a maximum voluntary effort and to sustain that effort to fatigue at the tension used in the study and with the various muscle groups examined here. Isometric training consisted of a series of brief (< 3 sec.) maximal voluntary contractions (MVC) with an intercontraction interval of 3 minutes. These were followed by a fatiguing isometric contraction. The tension exerted during the fatiguing contraction was set at 70% of the MVC. On any one day, only one direction of contraction was performed and all fatiguing contractions were held at the same percentage of isometric tension. This procedure was repeated on Monday, Wednesday, and Friday of successive weeks until, for any one muscle group (direction), the coefficient of variation (standard deviation divided by the mean) of endurance from day to day was reduced to less than 5%. In practice, the coefficient of variation of strength in these trained subjects was less than 3% from day to day by the end of the training period. Training was conducted at 70% MVC and with both muscle groups examined here. For most subjects, training for any one muscle group averaged about 3 weeks.

ISOMETRIC HEAD DYNAMOMETER

A helmet dynamometer has been developed which can be used to measure the strength and endurance of neck muscles in man in either one of four directions (forward flexion, backward extension, right and left lateral flexion). The dynamometer is based around the army SPH-4 type helmet, but

is easily adaptable to other types of military helmets as well. The dynamometer makes it possible to evaluate the effect of various types of dynamic activities and other flight activities on neck muscle strength and neck muscle endurance. It is, therefore, a useful tool in the study of military helmet design and evaluation of the stress induced by flight maneuvers. The isometric head dynamometer has been described in detail by Petrofsky and Phillips (1982) and is shown in Figures 2, 3 and 4.

HELMET SIMULATOR

The systematic assessment of significant helmet design parameters employed a helmet simulator in which both the weight and center-of-gravity were methodically and controllably altered. Such a helmet was developed by Simula, Inc., under subcontract to Wright State University.

The helmet simulator consists of two weight concealment boxes attached to opposite sides of a support ring (headring) which in turn is supported upon the wearer's head by a suspension system taken from an SPH-4 helmet. The weight and C.G. can be altered by positioning variable weights within the concealment boxes. Fabric covers over the boxes prevent the test subjects from obtaining visual clues as to the C.G. location.

The minimum weight of the helmet simulator, without any variable weights in the boxes, is 2.5 lb., slightly less than the weight of most quality crash helmets made by reputable manufacturers. The addition of variable weights to the boxes can alter the center of gravity to simulate the effect of equipment attached to the outside of a helmet. The helmet simulator has been calibrated for weights of 1.4, 1.8, 2.3, 3.2, and 4.1 kg. for each of the C.G. locations shown in Table 2. Figure 1 illustrates the range of C.G. variations together with definition of the coordinate axes by which the C.G. locations are measured.

As shown in Figure 5, a point midway between the left and right ear canals has been chosen as the origin of the coordinate axes. The helmet simulator is pictured in lateral (Fig. 6), oblique (Fig. 7) and frontal (Fig. 8) views. It has been provided with adjustment to ensure that an index point on it can be aligned with the ear canals, and also with independent adjustment to permit the suspension system to be made comfortable.

Eight headgear centers-of-gravity for three different headgear weights (a total of twenty-four headgear combinations) were evaluated (as per Tables 3 and 4) utilizing the variable center-of-gravity and variable weight helmet simulator. The "essential equivalency" between the variable center-of-gravity and variable weight helmet simulator and selected headgear loading configurations has been reported by Phillips and Petrofsky (1982b).

EXPERIMENTAL PROTOCOL

The experimental protocol may be summarized as follows:

Pre-Exercise MVC

With the subject seated in the helmet dynamometer, the subject would then either perform a brief (3 second) forward MVC (with EMG recorded from the sternocleidomastoid muscle) or a brief (3 second) right lateral MVC (with EMG recorded simultaneously from both the posterior neck muscles and sternocleidomastoid neck muscle). The contraction mode selected, would then be repeated at 3 minute intervals until 3 such contractions were performed.

The strongest contraction (highest strength and highest RMS amplitude of the EMG) would then be taken as the reference contraction.

Head Loading Configuration and Exercise Duration

With the subject removed from the isometric head dynamometer, alternating right and left lateral neck rotations were performed while wearing the variable center-of-gravity and variable weight helmet simulator which was set to one of the fifteen headgear combinations. The exercise duration was 30 minutes.

Post-Exercise Contractions

Immediately upon completion of the exercise period, the subject repositioned himself in the isometric head dynamometer, and a target tension of 70% of the pre-exercise MVC was sustained (in the direction of the pre-exercise MVC), and held to fatigue. The duration of this was called the endurance time.

The order of presentation of the direction of the pre-exercise MVC, the head loading configuration, and post-exercise contractions selected were all randomized for all of the subjects.

DEFINITION OF BOUNDARY CONDITIONS

The limits of displacement along the X- and Y-axes are shown in Figure 9 for the helmet simulator. The purpose of this section is to convert this frame of reference into a rectangular system of an equivalent area. The new X and Y coordinates are the boundary conditions for the computer model.

The area (T) of the shaded sections of Figure 9 is calculated as follows:

$$T = (.5)(.7)(1.8) + (.5)(.7)(1.8) + (2.5)(.7) + (.5)(.7)(1.8) + (.5)(.7)(1.8)$$

$$T = 0.63 + 0.63 + 1.75 + 0.63 + 0.63$$

$$T = 4.27\ cm^2$$

The total area (T^1) is the area of A and B plus T:

$$T^1 = (4.3 + 1.8)(1.8) + 4.27$$

$$T^1 = 10.98 + 4.27 = 15.25\ cm^2$$

Next define an incremental ΔX and ΔY such that:

$$(6.1 + 2\Delta X)(1.8 + \Delta Y) = 15.25$$

Simplify by defining:

$$\Delta X \equiv \Delta Y$$

Also, the boundaries are radial arcs, not straight lines, so that T is slightly (12%) larger than previously calculated:

$$T \equiv 4.76\ cm^2$$

So rewriting:

$$(6.1 + 2\Delta X)(1.8 + \Delta X) \equiv 15.74$$

Expanding:

$$10.98 + 6.1\Delta X + 3.6\Delta X + 2\Delta X^2 = 15.74$$

Rearranging:

$$(\Delta X)^2 + 4.85(\Delta X) - 2.38 = 0$$

Solving the quadratic for ΔX:

$$\Delta x = \frac{-4.85 \pm [(4.85)^2 - (4)(1)(-2.38)]^{\frac{1}{2}}}{(2)(1)}$$

$$\Delta X = \frac{-4.85 \pm [(23.52) + (9.52)]^{\frac{1}{2}}}{(2)}$$

$$\Delta X = \frac{-4.85 \pm 5.75}{2} = 0.45;\ -5.3$$

Since a negative distance is physically not realistic:

$$\Delta Y = \Delta X = 0.45$$

Check:

$$(6.1 + .9)(1.8 + .45) = 15.75$$

So that the rectangular x-y coordinate system is:

$$+X^1 = 4.3 + 0.45 = 4.75 \text{ cm}$$
$$-X^1 = 1.8 + 0.45 = 2.25 \text{ cm}$$
$$Y^1 = 1.8 + 0.45 = 2.25 \text{ cm}$$

which is shown in Figure 10.

TESTING OF ASSUMPTIONS

Referring once again to Figure 1, the important physical parameters of the system are: F, the load; M_X, the moment with respect to the X-axis; and M_Y, the moment with respect to the Y-axis.

Note that at these specific loading points, there is no moment with respect to the Z-axis (i.e., they are still parallel to it). That the Z-axis is not considered to be physiologically significant is why we chose point CH for our original study. In essence, the origin of the three axes ("O") sees the same F, whether at CL or at CH, and (of course) $M_X = M_Y = 0$. This was tested by observing whether forward and lateral contraction endurance times (for the neck muscles) with either the CH or CL configuration were similar (i.e., not statistically significantly different). Furthermore, we tested whether the effects of neck muscle loading were axi-symmetric (physiologically). That endurance times for C.G. displacement along the -Y axis (left side of the head/helmet) were similar to endurance times along the +Y axis (right side of the head/helmet) is why position LLL was selected in our final study. This was tested by observing whether forward and lateral neck muscle contraction endurance times were similar for either the LLL or RLL configuration (i.e., not statistically significantly different).

MATHEMATICAL METHODS

The model utilizes a three-dimensional space defined by the X, Y and F axes (see Fig. 11). The X-axis defines the location of the C.G. forward from the system origin to 4.75 cm anterior. The Y-axis defines the location of the C.G. lateral from the system origin to 2.25 cm right lateral. The F-axis defines the helmet configuration load from 1.45 kg. minimum weight to 4.09 kg. maximum weight.

The boundary conditions of the three-dimensional space are twelve endurance times (E_1 to E_{12}), which can be either the forward contraction or lateral contraction endurance time to sustain an isometric neck muscle contraction at 70% of the muscle's MVC. The boundary conditions are specified as $E_{Y,X,F}$, i.e., the endurance time after 30 minutes of dynamic exercise with a helmet C.G. located y cms along the Y-axis, x cms along the X-axis and a helmet weight of F kgs.

The three-dimensional system consists of eleven element faces (A through L) as shown at the bottom of Figure 11. Each element face requires a set of four subsidiary equations (1 to 4) and three primary equations (5 to 7) to describe the coordinate (E, endurance) point for that face. Face A is shown in Figure 12.

Face A equations are:

A1) $$E_{AF1} = E_2 + \left(\frac{F - 2.27}{1.82}\right)(E_3 - E_2)$$

A2) $$E_{AF2} = E_5 + \left(\frac{F - 2.27}{1.82}\right)(E_6 - E_5)$$

A3) $$E_{AY1} = E_2 + \left(\frac{Y}{2.25}\right)(E_5 - E_2)$$

A4) $$E_{AY2} + E_3 + \left(\frac{Y}{2.25}\right)(E_6 - E_3)$$

A5) $$E_{AF} = E_{AF1} + \left(\frac{Y}{2.25}\right)(E_{AF2} - E_{AF1})$$

A6) $$E_{AY} = E_{AY1} + \left(\frac{F - 2.27}{1.82}\right)(E_{AY2} - E_{AY1})$$

A7) $$E_A = \frac{E_{AF} + E_{AY}}{2}$$

Before we proceed to FACES B, C, D, F, G, H, I, J, K, L (in a similar manner), we repeat the process for the standard deviations (SD's).

The boundary conditions for the three dimensional space can also be the twelve standard deviations (SD_1 to SD_{12}) of the mean endurance times described previously. This is shown in Fig. 13 as a system of SD's ($S.D._{Y,X,F}$). The same eleven face elements (AA to LL) are present as with the endurance times. Each element face is described by the same set of seven equations. Face AA is shown in Fig. 14.

Face AA equations are:

AA1) $$SD_{AF1} = SD_2 + \left(\frac{F - 2.27}{1.82}\right)(SD_3 - SD_2)$$

AA2) $$SD_{AF2} = SD_5 + \left(\frac{F - 2.27}{1.82}\right)(SD_6 - SD_5)$$

AA3) $$SD_{AF2} = SD_2 + \left(\frac{Y}{2.25}\right)(SD_5 - SD_2)$$

AA4) $$SD_{AY2} = SD_3 + \left(\frac{Y}{2.25}\right)(SD_6 - SD_3)$$

AA5) $$SD_{AF} = SD_{AF1} + \left(\frac{Y}{2.25}\right)(SD_{AF2} - SD_{AF1})$$

AA6) $$SD_{AY} = SD_{AY1} + \left(\frac{F - 2.27}{1.82}\right)(SD_{AY2} - SD_{AY1})$$

AA7) $$SD_A = \frac{SD_{AF} + SD_{AY}}{2}$$

We now proceed to FACES B through L, but now computing an SD for each face (i.e., replace E with SD). Results are given in Appendix A.

For any load (F) between 2.27 kg. and 4.09 kg., a particular endurance time E can be interpolated using equations M1 to M4 as shown in Figure 15.

For any load (F) between 1.45 kgs. and 2.26 kgs., a particular endurance time E can be interpolated using equations M5 to M8 as shown in Figure 16.

Also recall since we have computed the SD's for each of the 11 faces:

MM1) $$SD_{AF} = SD_A + \left(\frac{X}{4.75}\right)(SD_F - SD_A)$$

MM2) $$SD_{CH} = SD_C + \left(\frac{Y}{2.25}\right)(SD_H - SD_C)$$

MM3) $SD_{KJ} = SD_K + \left(\frac{F - 2.27}{1.82}\right)(SD_J - SD_K)$

MM4) $SD = \frac{SD_{AF} + SD_{CH} + SD_{KJ}}{3}$

MM5) $SD_{BG} = SD_B + \left(\frac{X}{4.75}\right)(SD_G - SD_B)$

MM6) $SD_{DI} = SD_D + \left(\frac{Y}{2.25}\right)(SD_I - SD_D)$

MM7) $SD_{LK} = SD_L + \left(\frac{F - 1.45}{0.82}\right)(SD_K - SD_L)$

MM8) $SD = \frac{SD_{BG} + SD_{DI} + SD_{LK}}{3}$

The other half of the model utilizes a three-dimensional space defined by the -X, Y, and F axes (see Figure 17). The -X axis (or the W-axis) defines the location of the C.G. backward from the system origin to 2.25 cm rearward. The Y-axis and F-axis are as previously defined. The boundary conditions of the three-dimensional space are the four previous endurance times (E_1, E_3, E_4 and E_6) and eight more endurance times (E_{13} to E_{18}).

The three-dimensional system consists of eleven element faces (A and B previously, as well as N to V) as shown at the bottom of Figure 17. Each element face requires the same seven equations to describe the coordinate (E, endurance) point for that face. Note that Faces A and B were previously defined. Equations for Faces N to V are given in Appendix B. A system of SD's ($SD_{Y,W,F}$) can also be defined for face elements (NN to VV).

For any load (F) between 2.27 kg. and 4.09 kg., a particular endurance time E can be interpolated using equations W1 to W4 as shown in Figure 18.

For any load (F) between 1.45 kg. and 2.26 kg., a particular endurance time E can be interpolated using equations W5 to W8 as shown in Fig. 19.

Also recall since we have computed the SD's for each of the 11 faces:

WW1) $SD_{AP} = SD_A + \left(\frac{W}{2.25}\right)(SD_P - SD_A)$

WW2) $SD_{RN} = SD_R + \left(\frac{Y}{2.25}\right)(SD_N - SD_R)$

WW3) $SD_{UT} = SD_U + \left(\frac{F - 2.27}{1.82}\right)(SD_T - SD_U)$

WW4) $SD = \frac{SD_{AP} + SD_{RN} + SD_{UT}}{3}$

WW5) $SD_{BQ} = SD_B + (\frac{W}{2.25})(SD_Q - SD_B)$

WW6) $SD_{SO} = SD_S + (\frac{Y}{2.25})(SD_O - SD_S)$

WW7) $SD_{VU} = SD_V + (\frac{F - 1.45}{0.82})(SD_U - SD_V)$

WW8) $SD = \frac{SD_{BQ} + SD_{SO} + SD_{VU}}{3}$

We now modify the model to account for specific endurance times associated with forward or lateral contractions. Suppose for E_1 through E_{18}, these are the endurance times for forward contractions. Then define:

$$E = EF$$

and substitute this term in all the preceding equations, i.e., A1 through W8.

Suppose now that E_1 through E_{18} represent the respective endurance times for lateral contractions:

Then define:

$$E = EL$$

and substitute in all equations (A1 through W8).

Consequently, all equations (A1 through W8) must be written twice:

For example, regarding forward contractions:

A1) $EF_{AF1} = EF_2 + (\frac{F - 2.27}{1.82})(EF_3 - EF_2)$ to . . .

W8) $EF = \frac{EF_{BQ} + EF_{SO} + EF_{VU}}{3}$

For example, regarding lateral contractions:

A1) $EL_{AF1} = EL_2 + (\frac{F - 2.27}{1.82})(EL_3 + EL_2)$ to . . .

W8) $EL = \frac{EL_{BQ} + EL_{SO} + EL_{VU}}{3}$

The process can be easily repeated for the respective equations that solve for the standard deviation (SD).

For forward contractions, define:

SD = SF

and substitute this term in all the parallel equations (AA1 through WW8).

For lateral contractions, define:

SD = SL

and substitute this term in all the parallel equations (AA1 through WW8).

The computer program must be directed to solve for E and SD in one of four quadrants as shown in Figure 20.

This can be done by BASIC statements "IF...THEN..." which process the input data and direct the computer to the correct quadrant (i.e., sets of equations).

INPUT DATA (Range of Values):

```
F (Load):        1.45 kg to 4.09 kg
X (X-Axis):     -2.25 cm to 4.75 cm
YIN (Y-Axis):   -2.25 cm to 2.25 cm
```

DATA PREPROCESSING:

```
  60   Y = ABS (YIN)
  65  IF  F < 1.45  THEN  1000
  70  IF  F > 4.09  THEN  1000
  75  IF  Y > 2.25  THEN  1000
  80  IF  X > 4.75  THEN  1000
  85  IF  X < -2.25 THEN  1000
1000  PRINT "DATA OUT OF BOUNDS"
1005  END
```

DATA ROUTING

This is a series of tests to partition the data in the appropriate quadrant.

```
    100  IF  F < 2.27  THEN  300
    105  IF  X < 0  THEN  500
    110  REM:  Equations for Quadrant A:
               A1 - A7, C1 - C7, F1 - F7, H1 - H7,
               J1 - J7, K1 - K7, M1 - M4;
   184         AA1 - AA7, CC1 - CC7, FF1 - FF7,
Equations      HH1 - HH7, JJL - JJ7, KK1 - KK7,
               MM1 - MM4; (Equations written for EF, EL, SF, SL).
    290  GOTO 900
    300  IF X < 0  THEN  700
    305  REM:  Equations for Quadrant B:
               B1 - B7, D1 - D7, G1 - G7, I1 - I7,
   184         K1 - K7, L1 - L7, M5 - M8;
```

Equations BB1 - BB7, DD1 - DD7, GG1 - GG7,
II1 - II7, KK1 - KK7, LL1 - LL7,
MM5 - MM8; (Equations written for EF, EL, SF, SL)
490 GOTO 900
500 W = -X
505 REM: Equations for Quadrant C:
A1 - A7, N1 - N7, P1 - P7, R1 - R7,
184 T1 - T7, U1 - U7, W1 - W4;
Equations AA1 - AA7, NN1 - NN7, PP1 - PP7,
RR1 - RR7, TT1 - TT7, UU1 - UU7,
WW1 - WW4; (Equations written for EF, EL, SF, SL)
690 GOTO 900
700 W = -X
705 REM: Equations for Quadrant D:
B1 - B7, O1 - O7, Q1 - Q7, S1 - S7,
184 U1 - U7, V1 - V7, W5 - W8;
Equations BB1 - BB7, OO1 - OO7, QQ1 - QQ7,
SS1 - SS7, UU1 - UU7, VV1 - VV7,
WW5 - WW8; (Equations written for EF, EL, SF, SL)
900 PRINT "RESULTS ARE..."
995 GO TO 1005

The overall computer flow chart which results is shown in Figure 21.

RESULTS

The results are summarized into four categories: the endurance times (and their standard deviations) which represent the boundary conditions, the vertical endurance time, the horizontal endurance time, and the final computer model.

ENDURANCE TIME (BOUNDARY CONDITIONS)

The mean endurance times (and standard deviations) which were experimentally determined and used as the boundary conditions for the mathematical model are presented in Table 5. The stored constants used in the computer program (top of Figure 21), EF_1 to EF_{18} and SF_1 to SF_{18}, are those values presented in Table 5A. The other stored constants, EL_1 to EL_{18} and SL_1 to SL_{18}, are those values shown in Table 5B. This is summarized in Table 6.

ENDURANCE TIME (VERTICAL)

The endurance times (and standard deviations) for helmet weights of 1.45 kg., 2.27 kg. and 4.09 kg. when the C.G. was center-high (CH) compared to center-low (CL) are presented for forward isometric neck muscle contractions in Table 7A.

The endurance times (and standard deviations) for the same three helmet weights and same two C.G.'s are presented for lateral isometric neck muscle contractions in Table 7B.

For both tables, paired T-tests between the two C.G. locations (for each of the three helmet weights) showed no statistically significant difference.

ENDURANCE TIME (LATERAL)

The endurance times (and standard deviations) for helmet weights of 1.45 kg., 2.27 kg. and 4.07 kg. when the C.G. was left-lateral-low (LLL) compared to right-lateral-low (RLL) are presented for forward isometric neck muscle contractions in Table 8A.

The endurance times (and standard deviations) for the same three helmet weights and same two C.G.'s are presented for lateral isometric neck muscle contractions in Table 8B.

For both tables, paired T-tests between the two C.G. locations were not significantly different at 1.45 kg. and 2.27 kg. of helmet weight. The two C.G. locations were significantly different ($p \leq .05$) at the 4.09 kg. helmet weight.

COMPUTER MODEL

The final computer model was constructed utilizing the stored constants EF_1 - EF_{18}, SF_1 - SF_{18}, EL_1 - EF_{18} and SL_1 - SL_{18} (as per Table 6). EF_1 - EF_{18} were substituted into Eqs. A1 to W8 and EL_1 - EL_{18} were also substituted in Eqs. A1 to W8 as per the Mathematical Methods (see METHODS AND MATERIALS section). SF_1 - SF_{18} and SL_1 - SL_{18} were substituted into Eqs. AA1 to WW8. The program is user interactive and a complete listing is given in Appendix III.

The user enters the helmet weight (in kilograms), the X-axis coordinate of the C.G. and the Y-axis coordinate of the C.G. (in centimeters).

The program will then print out:

(a) the mean endurance time of a forward isometric neck muscle contraction (sustained at 70% of the MVC) in seconds;

(b) the standard deviation of (a);

(c) the mean endurance time of a lateral isometric neck muscle contraction (sustained at 70% of the MVC) in seconds;

(d) the standard deviation of (c).

A special significance test program has also been written and is listed in Appendix IV. If a second helmet weight and C.G. combination is also evaluated, the significance test program will test as to whether the first helmet weight and C.G loading combination is significantly diferent from the second helmet weight and C.G. loading combination with respect to neck muscle endurance times.

DISCUSSION AND CONCLUSIONS

A computer model of neck muscle endurance and fatigue as a function of helmet loading has been developed. The final model consists of over 700 equations and has been formatted to run on an Apple II+ and/or Apple IIe microcomputer with at least 48K of memory.

Our objective has been to make the model "definitive," but the final model deviates from this desired objective in several respects. An appreciation of those limitations will allow the user to more reasonably interpret results he/she may obtain from the model.

First, the model is valid only over a limited range of weight and C.G. locations. Specifically, helmet weight must be between 1.45 kg (3.2 lbs) and 4.09 kg (9.0 lbs), the X-axis displacement must be between -2.25 cm and 4.75 cm and the Y-axis displacement must be between -2.25 cm and +2.25 cm. Any conditions outside these limits will cause the program to print a "DATA OUT OF BOUNDS" statement.

Second, the model has been defined for a three-dimensional space based upon defined boundary conditions (see Methods and Materials, Mathematical Methods). It assumes that any helmet weight and C.G. location that falls within the three-dimensional space can be linearly interpolated between the various boundary conditions. In reality, the model is "piece-wise linear" between helmet weights of 1.45 kg to 2.26 kg and 2.27 kg to 4.09 kg. It is also "piece-wise" linear for X-axis displacements between -2.25 cm to 0 cm and 0 cm to 4.75 cm. Finally, it is "piece-wise" linear for Y-axis displacements between -2.25 cm to 0 cm and 0 cm to +2.25 cm.

Third, the assumption of insensitivity to vertical loading has been established, but only for the 0,0 X-Y coordinate. This assumption was not tested at any of the other boundary conditions. Furthermore, flight situations in which all G forces are not vertical, G_Z (i.e., where there is some lateral loading such as G_X or G_Y forces) may make the vertical insensitivity assumption invalid.

Fourth, the assumption of axi-symmetry (i.e., data aquired for right sided head loading is also valid for left sided head loading) has only been partially validated. The assumption appears valid for light-to-moderate helmet weights (i.e., 1.45 kg to 2.27 kg) but again was only tested at the 0,0 X-Y coordinate system. This assumption was not tested at any other boundary condition. Furthermore, for heavy helmet weights (4.09 kg) the assumption was not validated. Results predicted by the model for heavier weight helmets with left sided C.G. locations must be viewed with discretion.

Fifth and finally, laboratory based studies which predict isometric neck muscle endurance times (either forward or lateral) as a function of headgear loading cannot be extrapolated to predict "operational mission endurance times" (OMET). OMET's are a complex function of operator workload, heat, noise, vibration, etc.

Therefore, OMET's are multi-factorial problems which do not easily lend themselves to mathematical analysis. It was the inability of our group to translate our laboratory based isometric endurance times into meaningful OMET's that was a major factor in not pursuing this project further. We remain open to suggestions and would be pleased to submit a proposal in this area if warranted.

Subject	Age (yrs.)	Height (cm)	Weight (kg.)	Neck Cir. (cm)
1	22	180	77	37
2	28	175	85	37
3	22	193	79	38
4	20	188	73	37
5	20	185	70	36
6	19	168	61	36

TABLE 1. GENERAL CHARACTERISTICS OF THE SUBJECTS

C.G. Location Description*	Displacement (cm)**		
	X	Y	Z
Central			
Maximum Height	0	0	8.0
Medium Height	0	0	4.0
Low Height	0	0	0
Forward			
Maximum Height	5.0	0	8.0
Medium Height	5.0	0	4.0
Low Height	5.0	0	0
Aftward			
Maximum Height	-2.5	0	8.0
Medium Height	-2.5	0	4.0
Low Height	-2.5	0	0
Central			
Left, Maximum Height	0	2.5	8.0
Left, Medium Height	0	2.5	4.0
Left, Low Height	0	2.5	0
Central			
Right, Maximum Height	0	-2.5	8.0
Right, Medium Height	0	-2.5	4.0
Right, Low Height	0	-2.5	0
Forward			
Left, Maximum Height	4.3	1.8	8.0
Left, Medium Height	4.3	1.8	4.0
Left, Low Height	4.3	1.8	0
Forward			
Right, Maximum Height	4.3	-1.8	8.0
Right, Medium Height	4.3	-1.8	4.0
Right, Low Height	4.3	-1.8	0

* C.G. locations for total weights of less than 1.8 kg. may differ from values shown in this table.

** Displacement from head and neck c.g. (axis directions as defined in Figure 1).

TABLE 2. SPECIFIC C.G. LOCATIONS FOR WHICH THE HELMET SIMULATOR IS CALIBRATED

C.G.-Weight Description	Weight (kgs)	X	Displacement Y	(cm)* Z
CL				
Center-Low-3	1.45	0	0	0
Center-Low-5	2.27	0	0	0
Center-Low-9	4.09	0	0	0
CH				
Center-High-3	1.45	0	0	8.0
Center-High-5	2.27	0	0	8.0
Center-High-9	4.09	0	0	8.0
FL				
Forward-Low-3	1.45	5.0	0	0
Forward-Low-5	2.27	5.0	0	0
Forward-Low-9	4.09	5.0	0	0
LRL				
Lat-Right-Low-3	1.45	0	-2.5	0
Lat-Right-Low-5	2.27	0	-2.5	0
Lat-Right-Low-9	4.09	0	-2.5	0
AL				
Afterward-Low-3	1.45	-2.5	0	0
Afterward-Low-5	2.27	-2.5	0	0
Afterward-Low-9	4.09	-2.5	0	0

* Displacement from the head/neck center-of-gravity: axis directions as per Fig. 1, and displacement distances as per Table 2.

TABLE 3. FIFTEEN HEADGEAR COMBINATIONS ORIGINALLY EVALUATED

C.G.-Weight Description	Weight (kgs)	X	Displacement Y	(cm)* Z
FRL				
For-Right-Low-3	1.45	4.3	-1.8	0
For-Right-Low-5	2.27	4.3	-1.8	0
For-Right-Low-9	4.09	4.3	-1.8	0
ARL				
Aft-Right-Low-3	1.45	-1.8	-1.8	0
Aft-Right-Low-5	2.27	-1.8	-1.8	0
Aft-Right-Low-9	4.09	-1.8	-1.8	0
LLL				
Lat-Left-Low-3	1.45	0	2.5	0
Lat-Left-Low-5	2.27	0	2.5	0
Lat-Left-Low-9	4.09	0	2.5	0

* Same as footnote (Table 3.)

TABLE 4. NINE ADDITIONAL HEADGEAR COMBINATIONS EVALUATED

A. Forward Contractions

Wt. / *C.G.	1.45 kg	2.27 kg.	4.09 kg.
FRL	49 ± 19	37 ± 10	41 ± 9
FL	86 ± 49	65 ± 26	56 ± 24
LRL	75 ± 57	66 ± 21	96 ± 61
CL	98 ± 34	74 ± 43	68 ± 34
ARL	43 ± 12	46 ± 16	45 ± 13
AL	86 ± 34	64 ± 19	83 ± 20

B. Lateral Contractions

Wt. / *C.G.	1.45 kg.	2.27 kg.	4.09 kg.
FRL	105 ± 25	102 ± 26	122 ± 49
FL	148 ± 53	89 ± 42	62 ± 35
LRL	156 ± 59	81 ± 40	72 ± 40
CL	106 ± 88	83 ± 33	86 ± 38
ARL	82 ± 31	79 ± 47	94 ± 31
AL	91 ± 38	73 ± 47	129 ± 69

* See "CG-Weight Description" Column (Table 3).

TABLE 5. ENDURANCE TIME: BOUNDARY CONDITIONS (Secs)

CONSTANT	VALUE	CONSTANT	VALUE	CONSTANT	VALUE	CONSTANT	VALUE
EF_1	98	SF_1	34	EL_1	106	SL_1	88
EF_2	74	SF_2	43	EL_2	83	SL_2	33
EF_3	68	SF_3	34	EL_3	86	SL_3	38
EF_4	75	SF_4	57	EL_4	156	SL_4	59
EF_5	66	SF_5	21	EL_5	81	SL_5	40
EF_6	96	SF_6	61	EL_6	72	SL_6	40
EF_7	49	SF_7	19	EL_7	105	SL_7	25
EF_8	37	SF_8	10	EL_8	102	SL_8	26
EF_9	41	SF_9	9	EL_9	122	SL_9	49
EF_{10}	86	SF_{10}	49	EL_{10}	148	SL_{10}	53
EF_{11}	65	SF_{11}	26	EL_{11}	89	SL_{11}	42
EF_{12}	56	SF_{12}	24	EL_{12}	62	SL_{12}	35
EF_{13}	43	SF_{13}	12	EL_{13}	82	SL_{13}	31
EF_{14}	46	SF_{14}	16	EL_{14}	79	SL_{14}	47
EF_{15}	45	SF_{15}	13	EL_{15}	94	SL_{15}	31
EF_{16}	86	SF_{16}	34	EL_{16}	91	SL_{16}	38
EF_{17}	64	SF_{17}	19	EL_{17}	73	SL_{17}	47
EF_{18}	83	SF_{18}	20	EL_{18}	129	SL_{18}	69

TABLE 6. IDENTIFICATION OF STORED CONSTANTS

A. Forward Contractions

*C.G. \ Wt.	1.45 kg.	2.27 kg.	4.09 kg.
CH	71 ± 49	82 ± 59	71 ± 55
CL	98 ± 34	74 ± 43	68 ± 34
p-level	N.S.	N.S.	N.S.

B. Lateral Contractions

*C.G. \ Wt.	1.45 kg.	2.27 kg.	4.09 kg.
CH	82 ± 56	72 ± 35	61 ± 39
CL	106 ± 88	83 ± 33	86 ± 38
p-level	N.S.	N.S.	N.S.

* See footnote (Table 5).

TABLE 7. ENDURANCE TIME: VERTICAL (Secs)

A. Forward Contractions

Wt. / *C.G.	1.45 kg.	2.27 kg.	4.09 kg.
LLL	46 ± 18	47 ± 19	43 ± 10
RLL	75 ± 57	66 ± 21	96 ± 61
p-level	N.S.	N.S.	$p \leq .05$

B. Lateral Contractions

Wt. / *C.G.	1.45 kg.	2.27 kg.	4.09 kg.
LLL	108 ± 50	110 ± 27	116 ± 35
RLL	156 ± 59	81 ± 40	72 ± 40
p-level	N.S.	N.S.	$p \leq .05$

* See footnote (Table 7).

TABLE 8. ENDURANCE TIME: LATERAL (Secs)

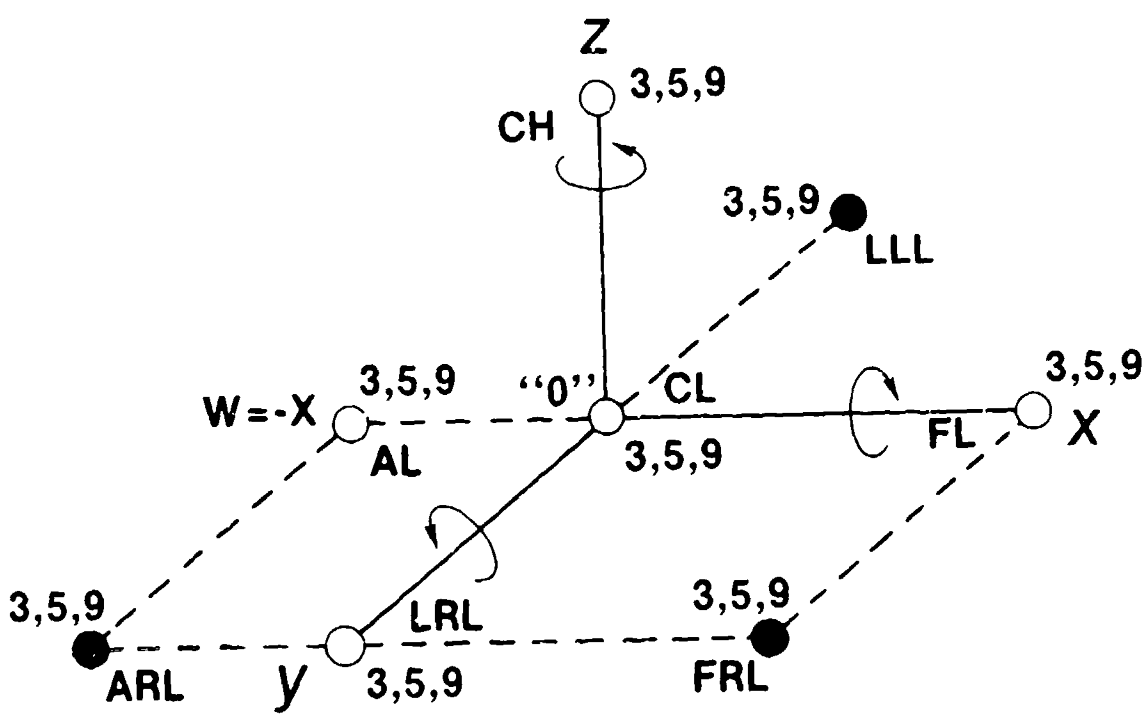


FIGURE 1. THE EIGHT C.G. LOCATIONS EVALUATED AT EACH OF THREE WEIGHTS

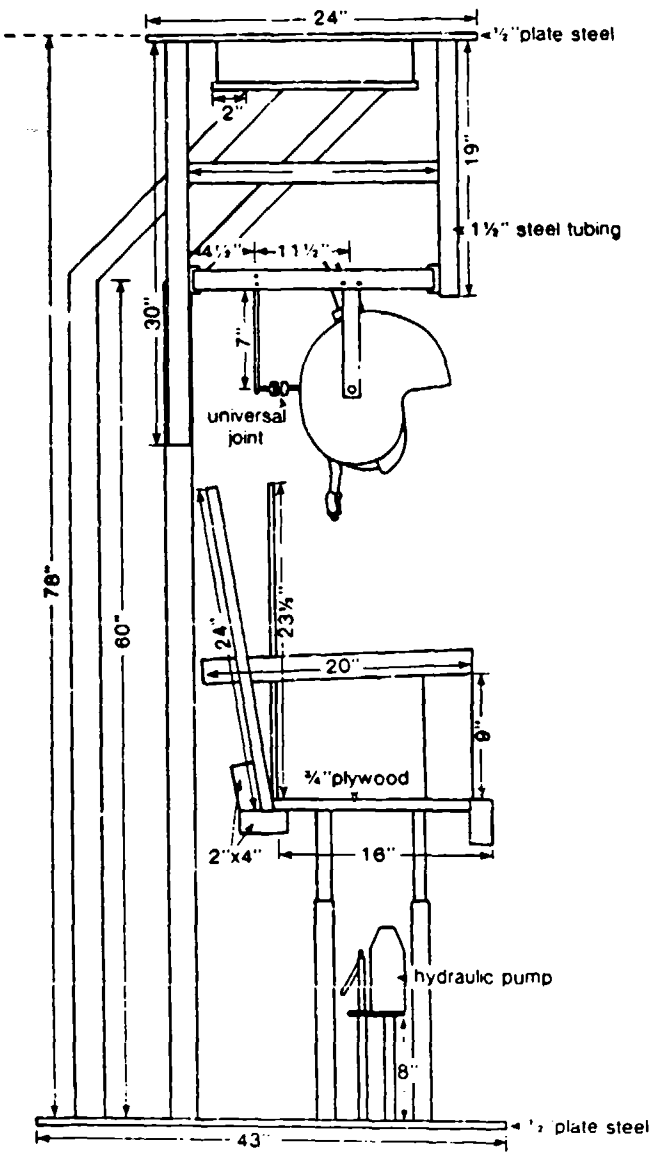


FIGURE 2. SCHEMATIC DIAGRAM OF THE HEAD DYNAMOMETER

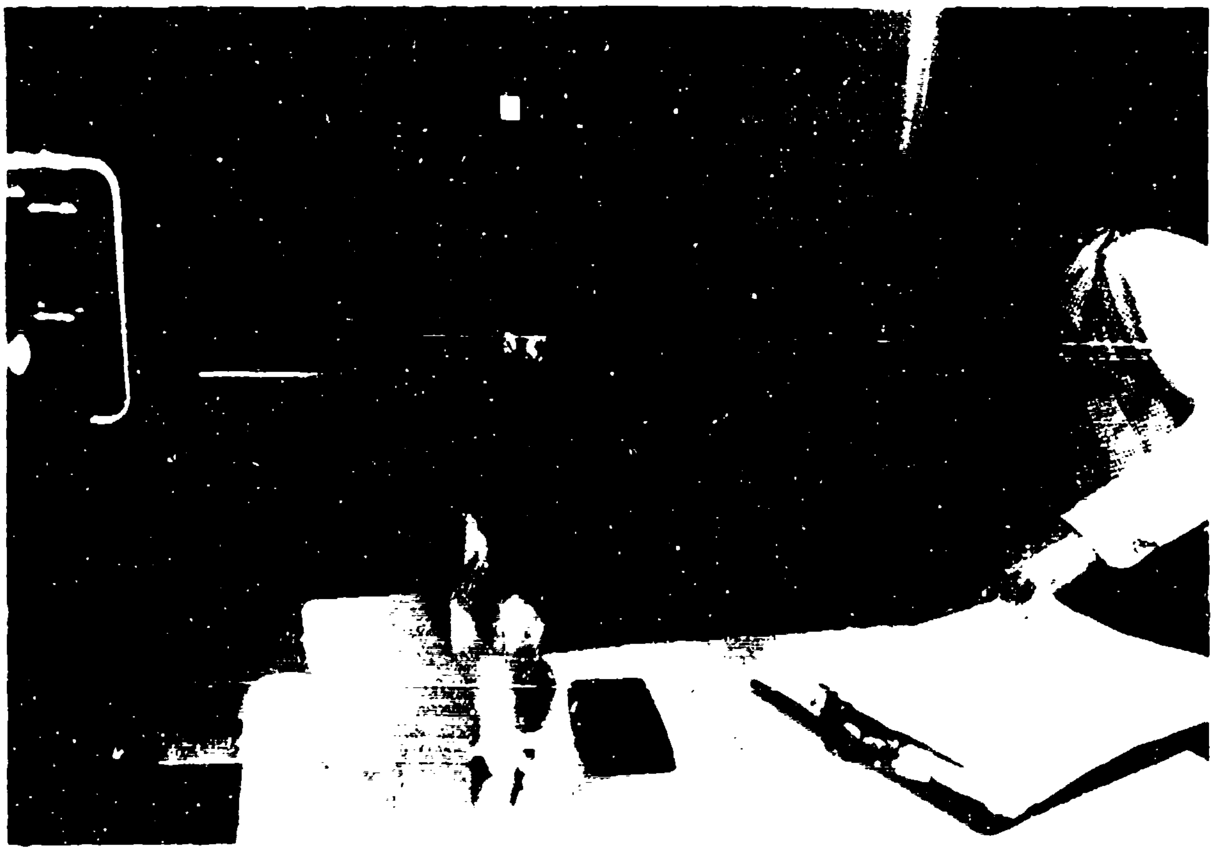


FIGURE 3. SUBJECT USING THE HEAD DYNAMOMETER: FRONTAL VIEW

FIGURE 4. SUBJECT USING THE HEAD DYNAMOMETER: LATERAL VIEW

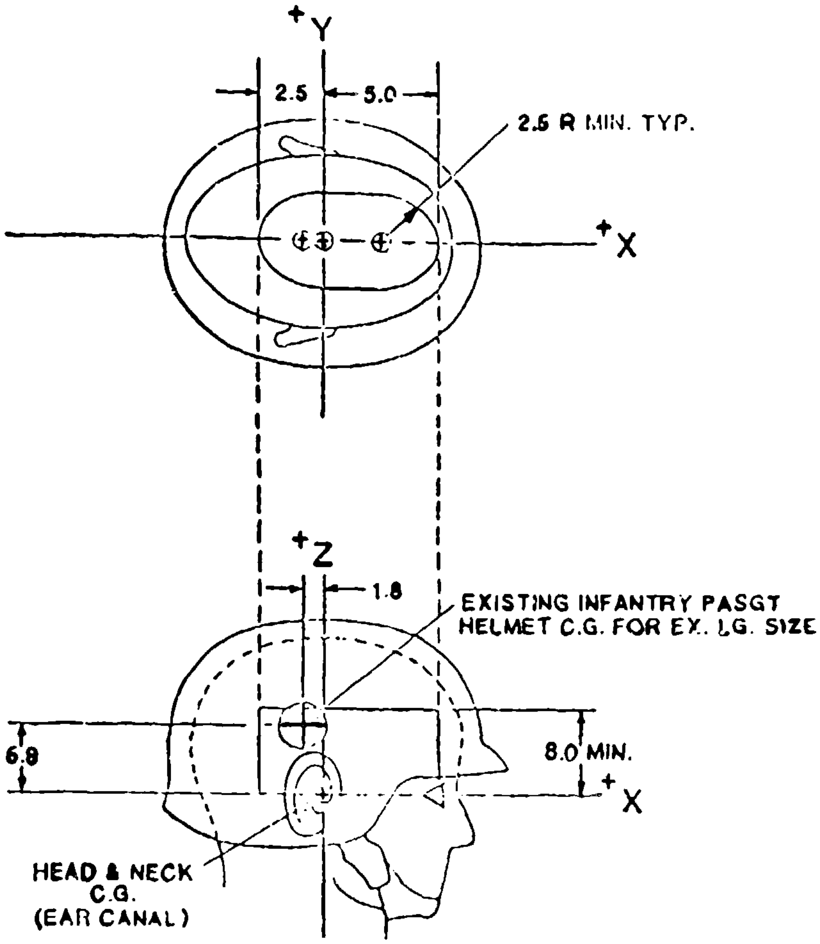


FIGURE 5. COORDINATE SYSTEM FOR THE HELMET SIMULATOR

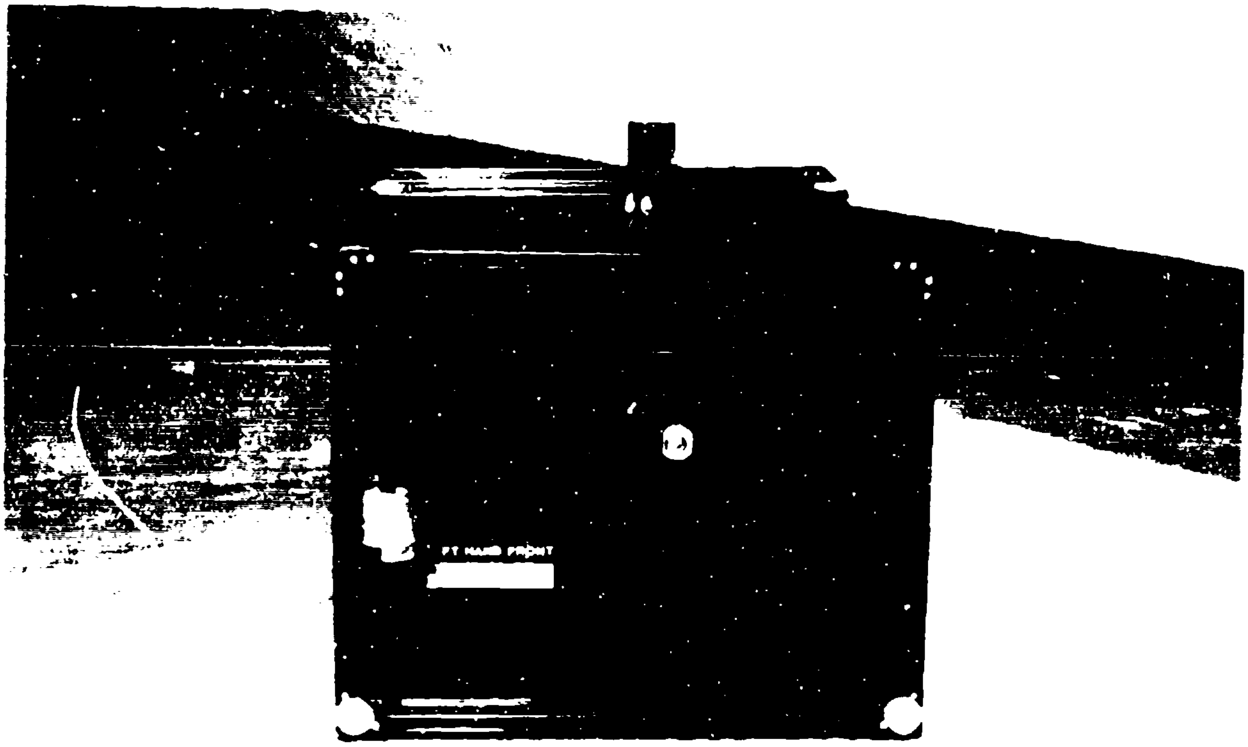


FIGURE 6. HELMET SIMULATOR: LATERAL VIEW

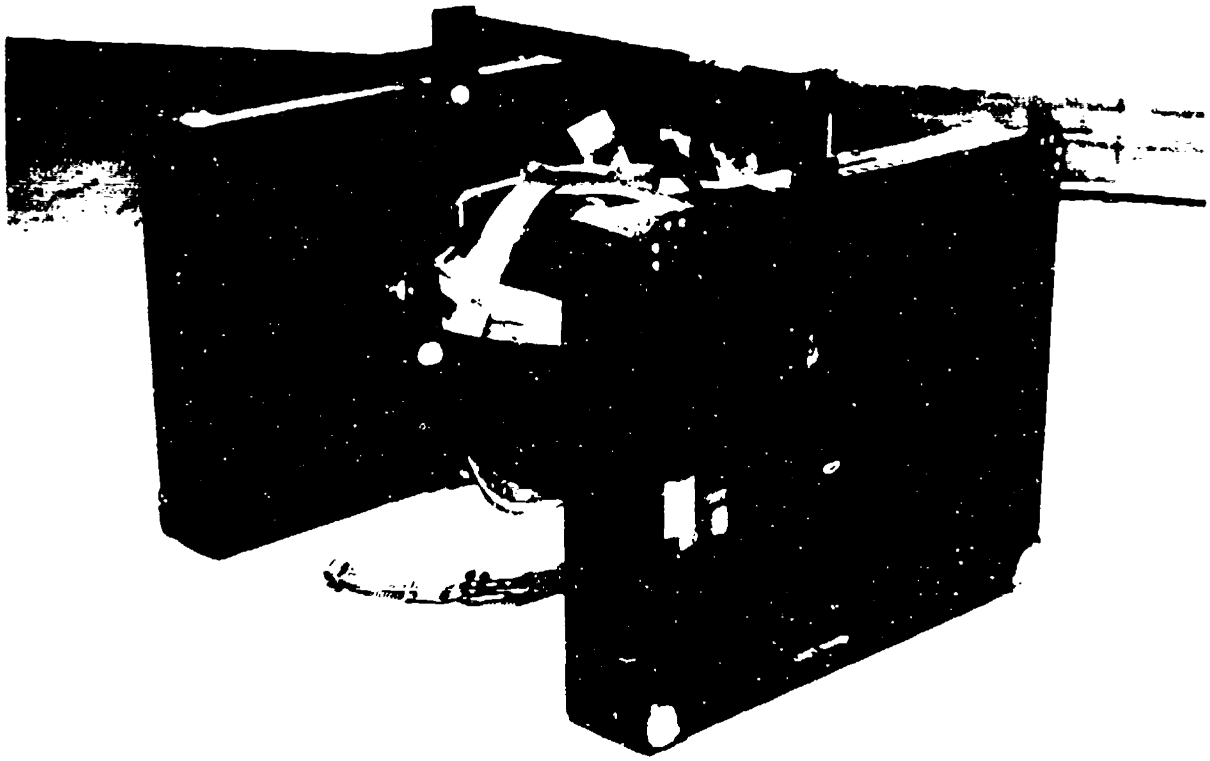


FIGURE 7. HELMET SIMULATOR: OBLIQUE VIEW

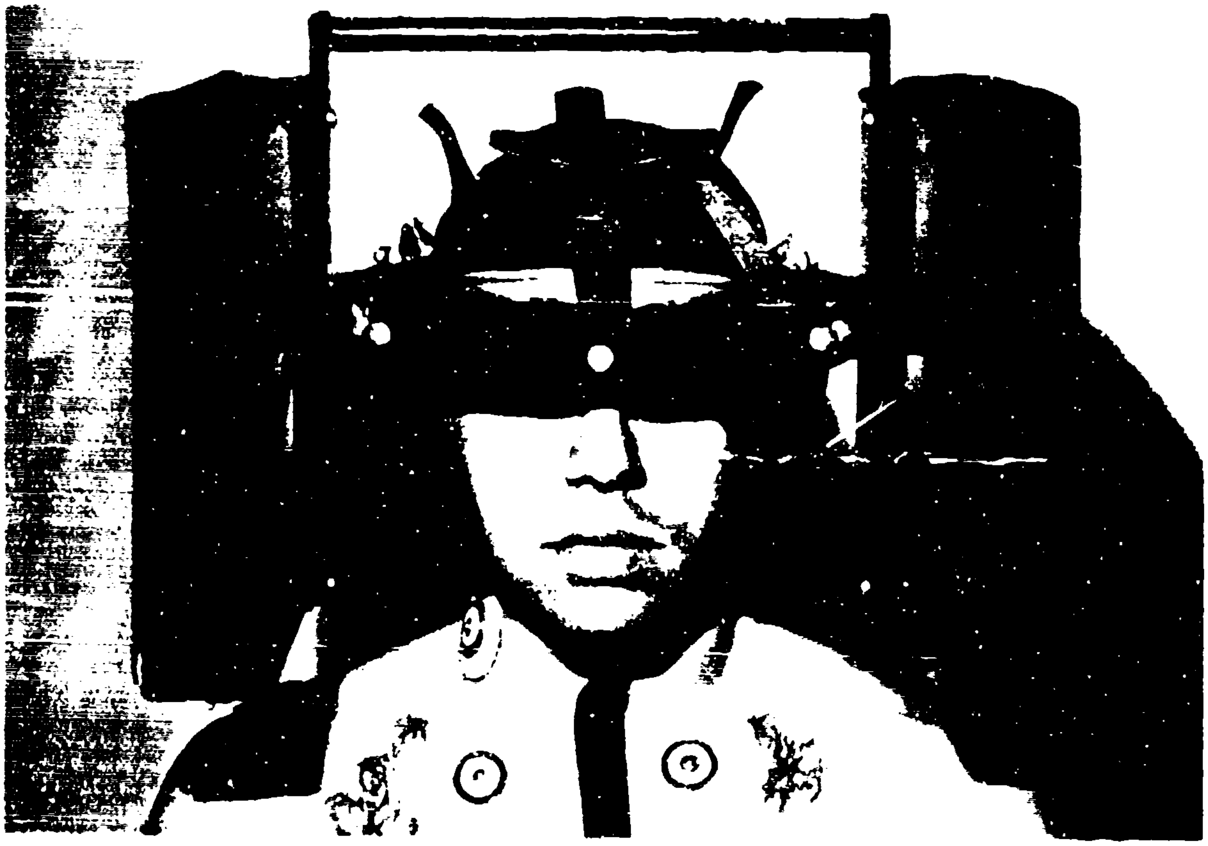


FIGURE 8. SUBJECT USING THE HELMET SIMULATOR: FRONTAL VIEW

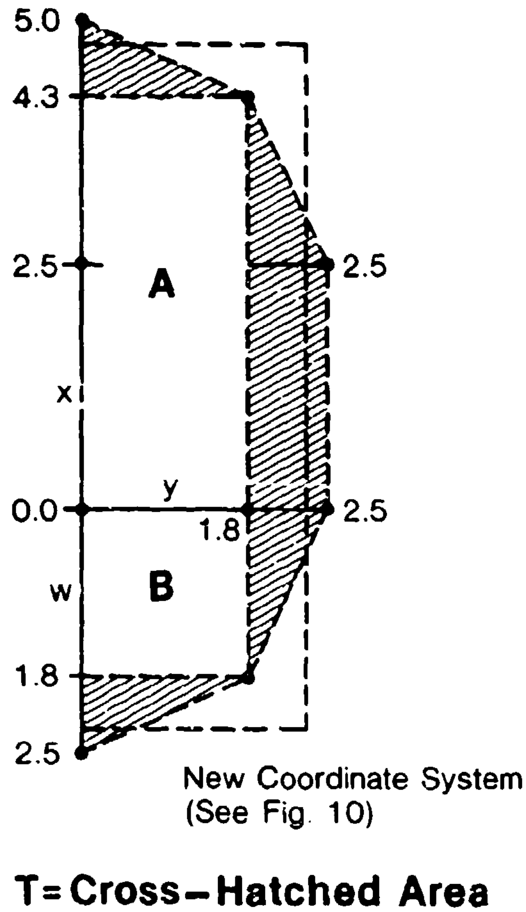


FIGURE 9. SCHEMATIC OF THE ACTUAL HELMET SIMULATOR COORDINATE SYSTEM

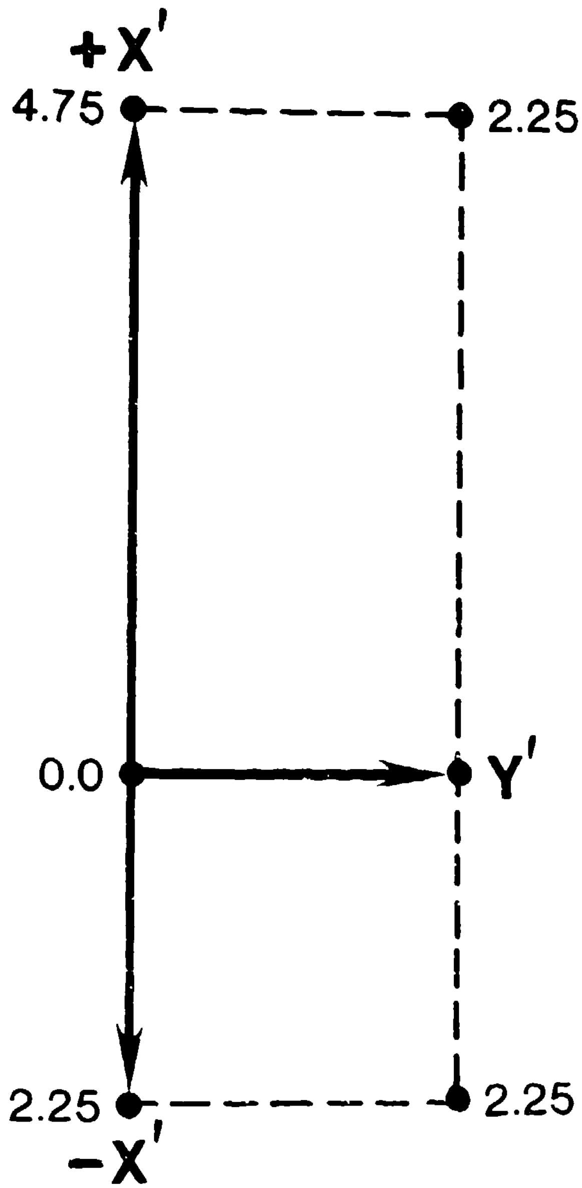
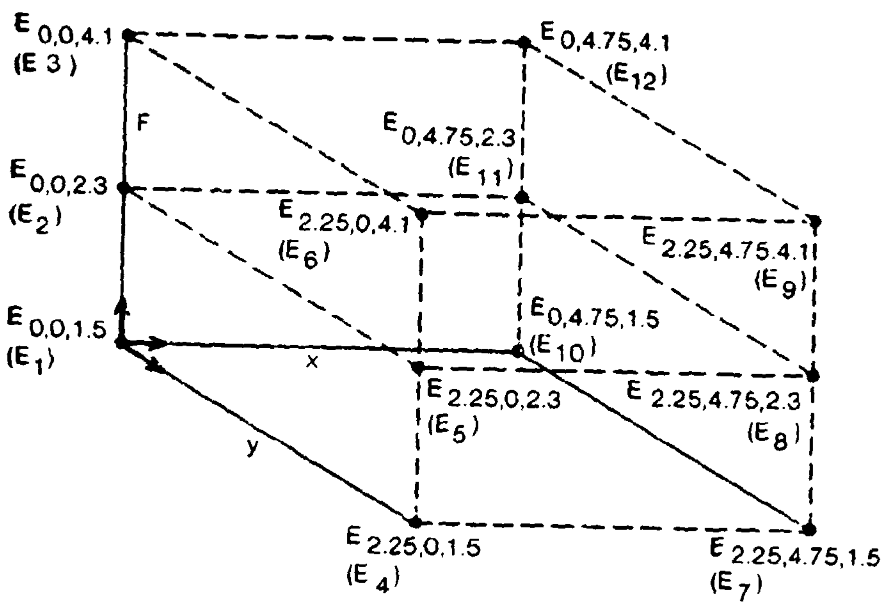


FIGURE 10. THE SIMPLIFIED HELMET SIMULATOR COORDINATE SYSTEM

Three Dimensional System: (Ey,x,f)



Simplified: (see also SD's)

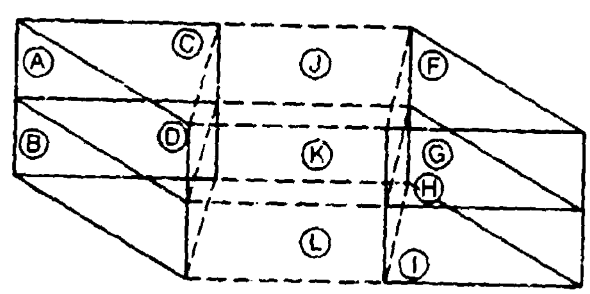


FIGURE 11. THE THREE DIMENSIONAL SPACE SYSTEM ($E_{y,x,f}$)

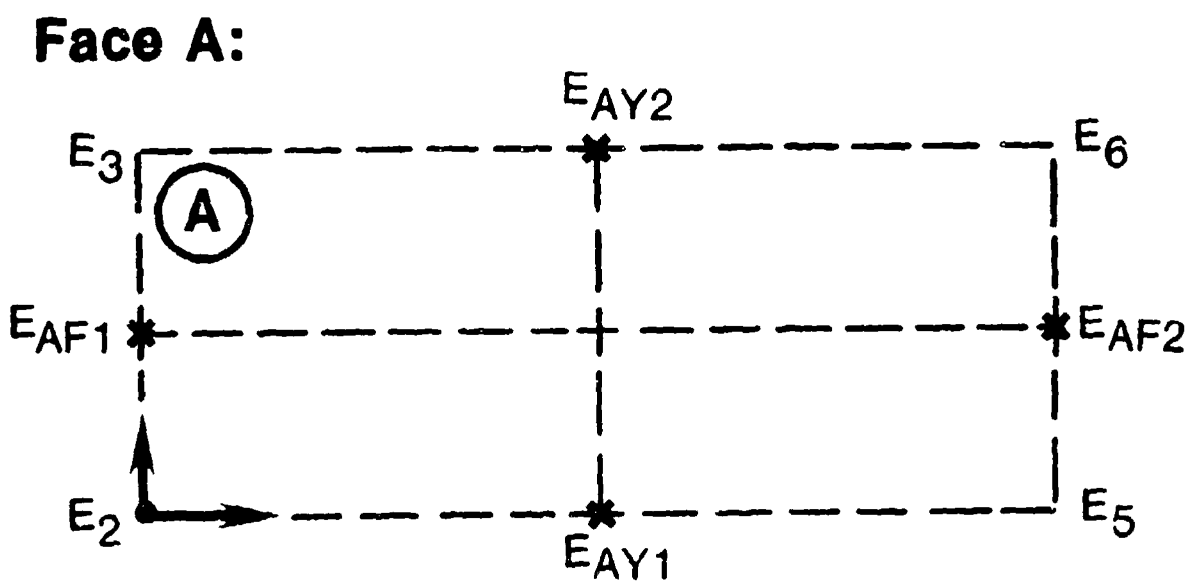


FIGURE 12. FACE "A" ELEMENT

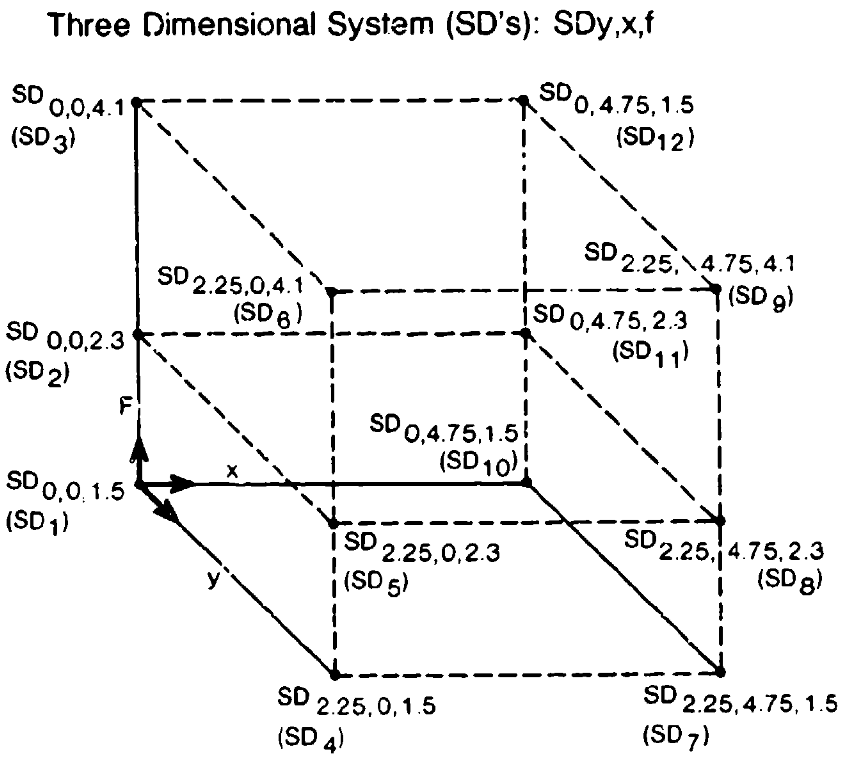


FIGURE 13. THE THREE DIMENSIONAL SPACE SYSTEM ($SD_{y,x,f}$)

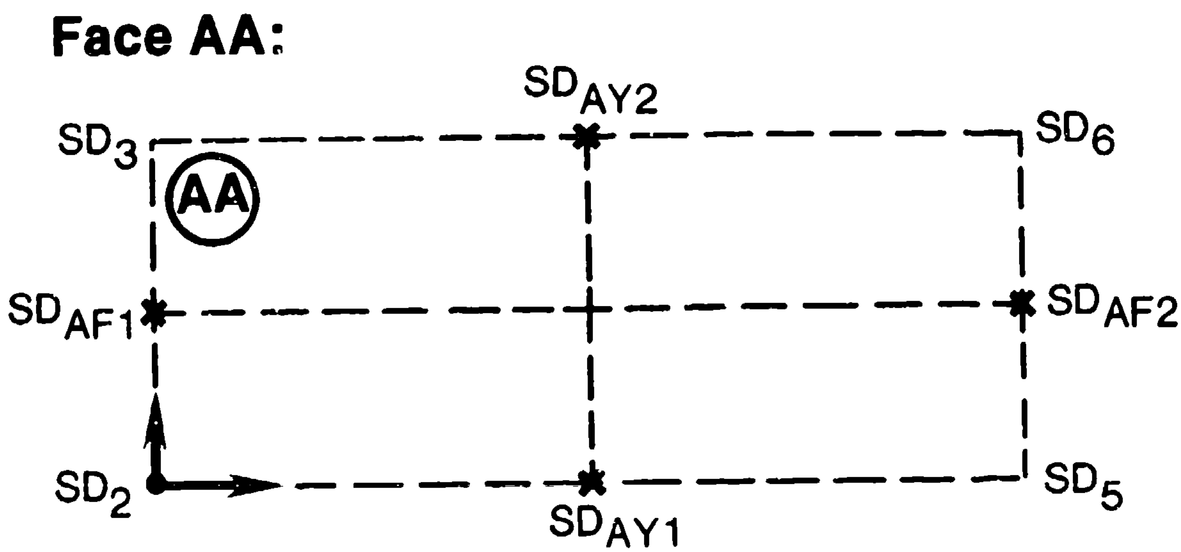
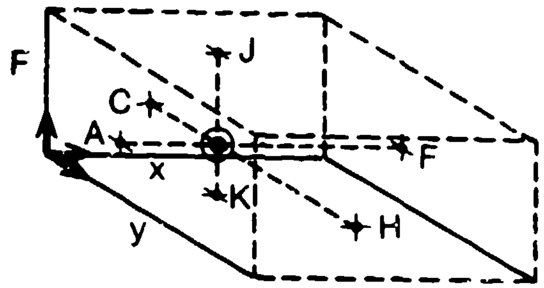


FIGURE 14. FACE "AA" ELEMENT

For F = 2.27 to 4.09 Kg



$$\text{M1)}\quad E_{AF} = E_A + \left(\frac{x}{4.75}\right)(E_F - E_A)$$

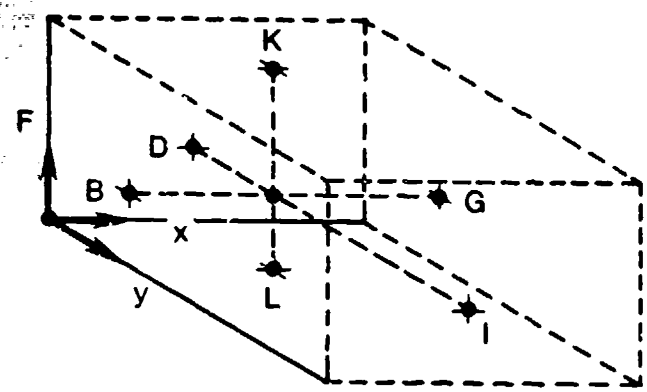
$$\text{M2)}\quad E_{CH} = E_C + \left(\frac{y}{2.25}\right)(E_H - E_C)$$

$$\text{M3)}\quad E_{KJ} = E_K + \left(\frac{F-2.27}{1.82}\right)(E_J - E_K)$$

$$\text{M4)}\quad E = \frac{E_{AF} + E_{CH} + E_{KJ}}{3}$$

FIGURE 15. QUADRANT 'A' SPACE SYSTEM AND EQUATIONS

For F = 1.45 to 2.26 Kg



$$\text{M5)}\quad E_{BG} = E_B + \left(\frac{x}{4.75}\right)(E_G - E_B)$$

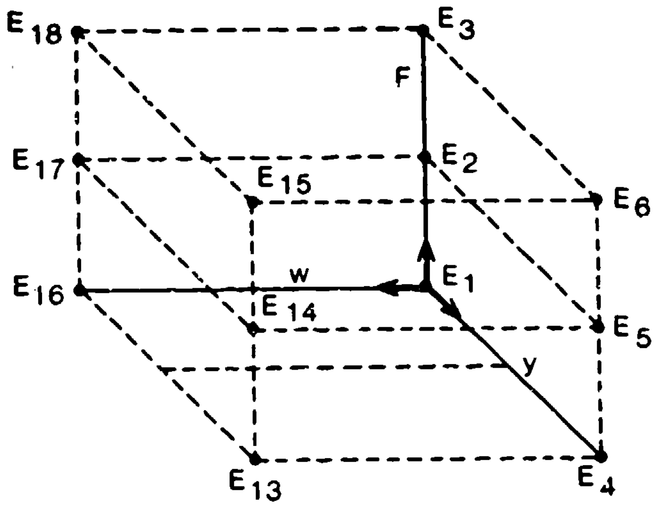
$$\text{M6)}\quad E_{DI} = E_D + \left(\frac{y}{2.25}\right)(E_I - E_D)$$

$$\text{M7)}\quad E_{LK} = E_L + \left(\frac{F-1.45}{0.82}\right)(E_K - E_L)$$

$$\text{M8)}\quad E = \frac{E_{BG} + E_{DI} + E_{LK}}{3}$$

FIGURE 16. QUADRANT 'B' SPACE SYSTEM AND EQUATIONS

Three Dimensional System: (E_y,w,f)



In this system w = -x

Simplified:

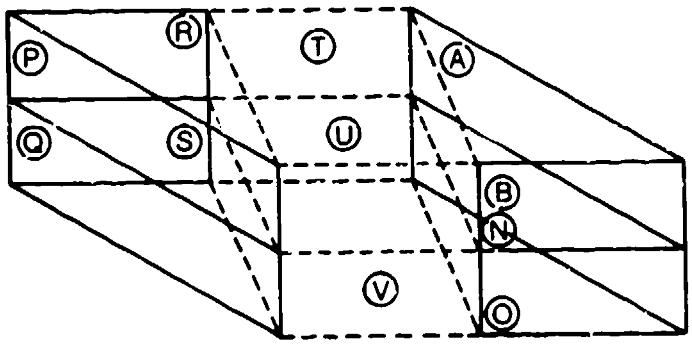
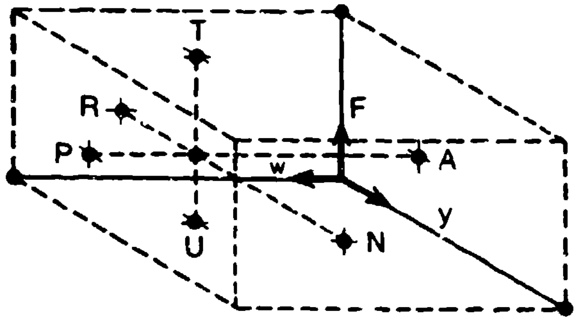


FIGURE 17. THE THREE DIMENSIONAL SPACE SYSTEM ($E_{y,w,f}$)

For F = 2.27 to 4.09 Kg



$$W1)\quad E_{AP} = E_A + \left(\frac{w}{2.25}\right)(E_P - E_A)$$

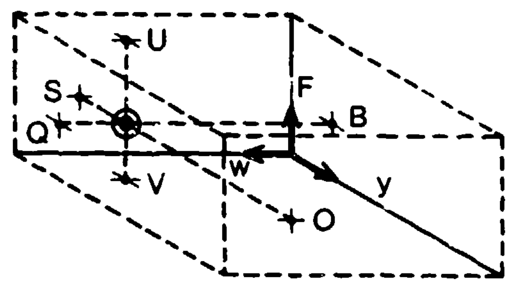
$$W2)\quad E_{RN} = E_R + \left(\frac{y}{2.25}\right)(E_N - E_R)$$

$$W3)\quad E_{UT} = E_U + \left(\frac{F\;\;2.27}{1.82}\right)(E_T - E_U)$$

$$W4)\quad E = \frac{E_{AP} + E_{RN} + E_{UT}}{3}$$

FIGURE 18. QUADRANT 'C' SPACE SYSTEM AND EQUATIONS

For F=1.45 to 2.26 Kg



$$W5)\quad E_{BQ} = E_B + \left(\frac{w}{2.25}\right)(E_Q - E_B)$$

$$W6)\quad E_{SO} = E_S + \left(\frac{y}{2.25}\right)(E_O - E_S)$$

$$W7)\quad E_{VU} = E_V + \left(\frac{F-1.45}{0.82}\right)(E_U - E_V)$$

$$W8)\quad E = \frac{E_{BQ} + E_{SO} + E_{VU}}{3}$$

FIGURE 19. QUADRANT 'D' SPACE SYSTEM AND EQUATIONS

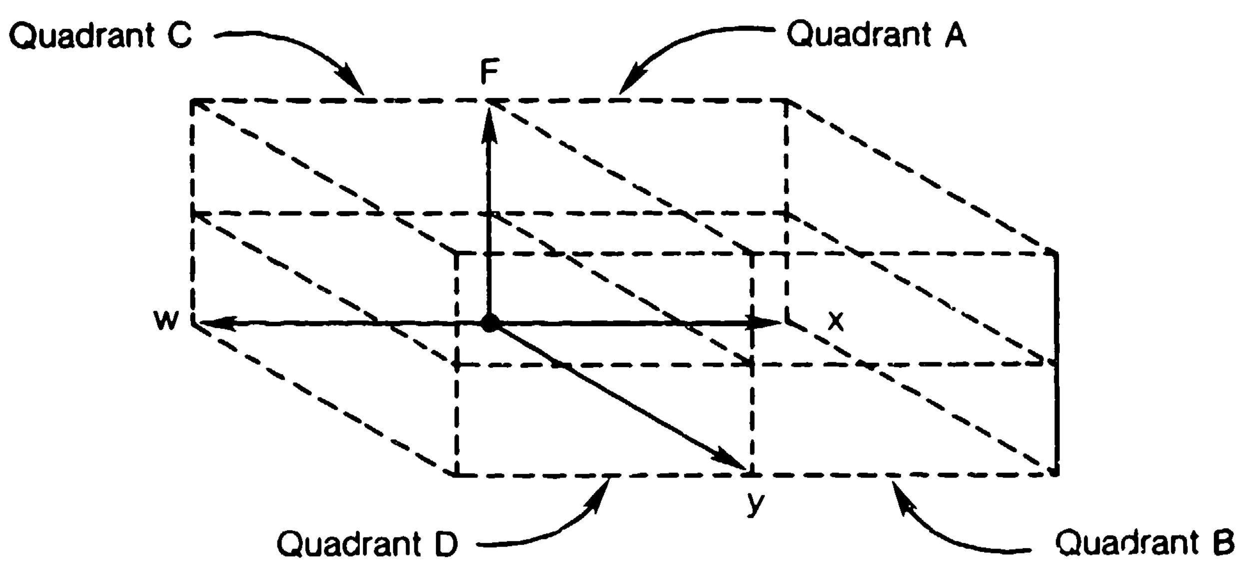


FIGURE 20. SUMMARY OF THE FOUR QUADRANT SYSTEM

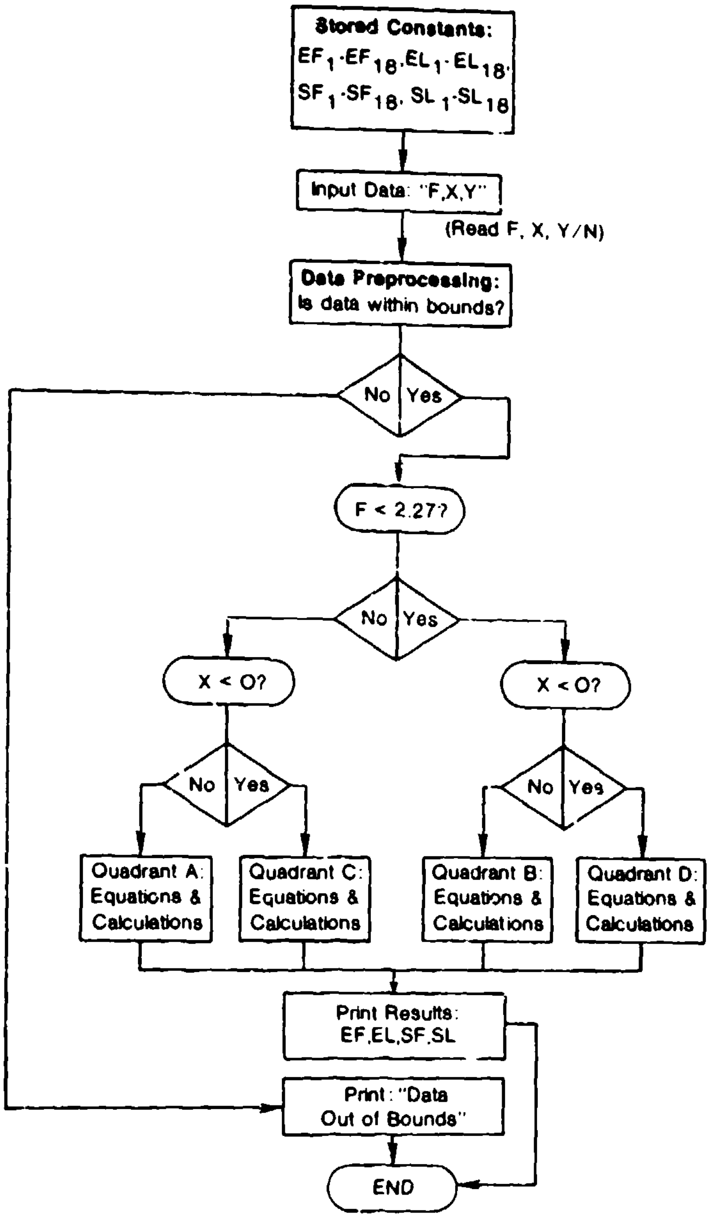


FIGURE 21. FLOW DIAGRAM OF THE COMPUTER PROGRAM

REFERENCES CITED

*Petrofsky, J.S. and C.A. Phillips. 1982. The Strength-Endurance Relationship in Skeletal Muscle: Its Application to Helmet Design. Aviat. Space Environ. Med. 53: 365-369.

*Phillips, C.A. and J.S. Petrofsky. 1981a. The Strength-Endurance Relationship of the Neck Muscles: A Physiological Basis for Helmet Design. Proc. Aerospace Med. Assoc. 81: 217-218.

*Phillips, C.A. and J.S. Petrofsky. 1981b. Physiological Criteria for Helmet Design. Federation Proc. 40: 497.

*Phillips, C.A. and J.S. Petrofsky. 1981c. A Quantitative Evaluation of Neck Muscle Performance. The Physiologist. 24: 66.

*Phillips, C.A. and J.S. Petrofsky. 1981d. Influence of U.S. Army Headgear Parameters on Neck Muscle Loading and Fatigue. Annual Report. Army contract DAMD17-80-C-0089. U.S. Army Medical Research and Development Command.

*Phillips, C.A. and J.S. Petrofsky. 1982a. Electromyographic Changes in the Lateral and Posterior Neck Muscles Associated with Conventional Helmet Loading. Proc. Aerospace Med. Assoc. 82: 59-60.

*Phillips, C.A. and J.S. Petrofsky. 1982b. Validation Study of the VCGW Helmet Simulator. Proc. 35th ACEMB. 24: 15.

*Phillips, C.A. and J.S. Petrofsky. 1982c. Cardiovascular Response to Neck Muscle Exertion. The Physiologist. 25: 202.

*Phillips, C.A. and J.S. Petrofsky. 1982d. Systematic Variation of U.S. Army Headgear Parameters and Neck Muscle Loading and Fatigue. Annual Report. Army contract DAMD17-80-C-0089. U.S. Army Medical Research and Development Command.

*Phillips, C.A. and J.S. Petrofsky. 1983a. Quantitative Electromyography: Response of the Neck Muscles to Conventional Helmet Loading. Aviat. Space Environ. Med. 54: 452-457.

*Phillips, C.A. and Petrofsky, J.S. 1983b. Neck Muscle Loading and Fatigue: Systematic Variation of Headgear Weight and Center-of-Gravity. Aviat. Space Environ. Med. 54: 901-905.

*Phillips, C.A. and J.S. Petrofsky. 1983c. Influence of Variable Headgear Loading on Blood Pressure and Heart Rate. The Physiologist. 26: A-112.

*Phillips, C.A. and J.S. Petrofsky. 1984a. Neck Muscle Endurance and Fatigue as a Function of Helmet Loading: The Definitive Mathematical Model. Sci. Prog., 55th Ann. Sci. Meet., Aero. Med. Assoc., : A19.

*Phillips, C.A. and J.S. Petrofsky. 1984b. Cardiovascular Responses During Fatiguing Isometric Contractions of the Neck Muscles. Aviat. Space Environ. Med. 55:740-745.

*Phillips, C.A. and J.S Petrofsky. 1986. A Computer Model of Neck Muscle Endurance and Fatigue as a Function of Helmet Loading. Comput. Biol. Med. 16:103-130.

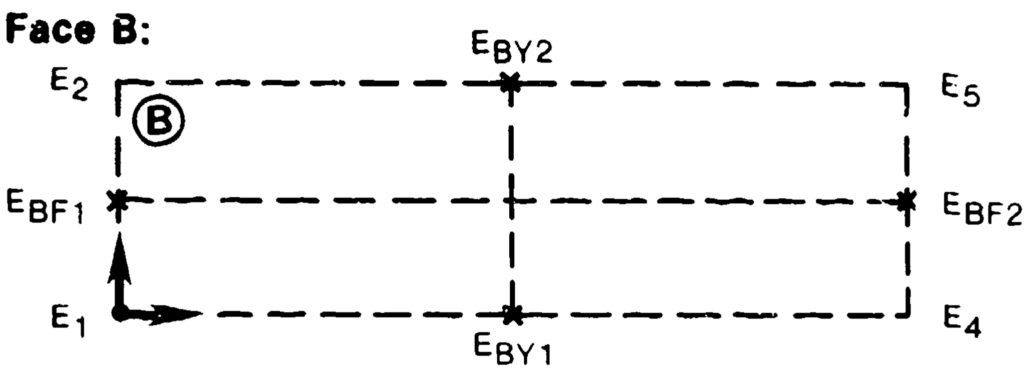
*Publications supported by contract no. DAMD17-80-C-0089.

DISTRIBUTION LIST

1 copy	Commander US Army Medical Research and Development Command ATTN: SGRD-RMI-S Fort Detrick, Frederick, MD 21701-5012
12 copies	Defense Technical Information Center (DTIC) ATTN: DTIC-DDAC Cameron Station Alexandria, VA 22304-6145
1 copy	Dean School of Medicine Uniformed Services University of the Health Sciences 4301 Jones Bridge Road Bethesda, MD 20814-4799
1 copy	Commandant Academy of Health Sciences, US Army ATTN: AHS-CDM Fort Sam Houston, TX 78234-6100
1 copy	Commander U.S. Army Aeromedical Research Laboratory ATTN: Dr. Kent Kimball/SGRD-UAB Fort Rucker, AL 36362-5000

APPENDIX A

SUBSIDIARY EQUATIONS (FACE ELEMENTS B-L)



B1) $E_{BF1} = E_1 + \left(\frac{F - 1.45}{0.82}\right)(E_2 - E_1)$

B2) $E_{BF2} = E_4 + \left(\frac{F - 1.45}{0.82}\right)(E_5 - E_4)$

B3) $E_{BY1} = E_1 + \left(\frac{Y}{2.25}\right)(E_4 - E_1)$

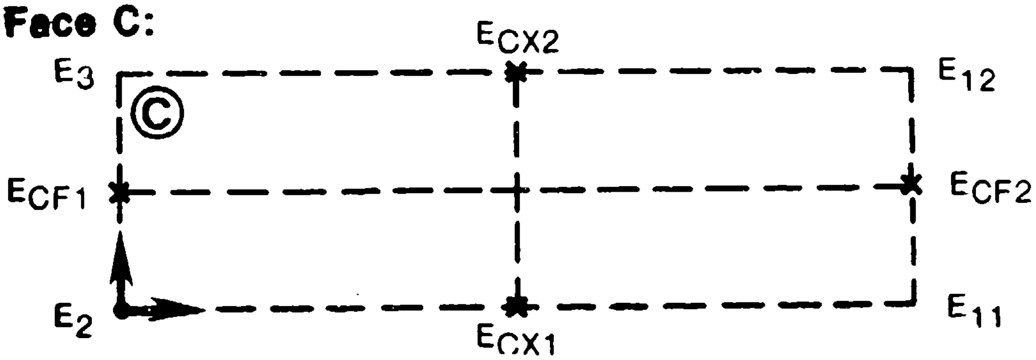
B4) $E_{BY2} = E_2 + \left(\frac{Y}{2.25}\right)(E_5 - E_2)$

B5) $E_{BF} = E_{BF1} + \left(\frac{Y}{2.25}\right)(E_{BF2} - E_{BF1})$

B6) $E_{BY} = E_{BY1} + \left(\frac{F - 1.45}{0.82}\right)(E_{BY2} - E_{BY1})$

B7) $E_B = \frac{E_{BF} + E_{BY}}{2}$

Face C:



C1) $E_{CF1} = E_2 + \left(\frac{F - 2.27}{1.82}\right)(E_3 - E_2)$

C2) $E_{CF2} = E_{11} + \left(\frac{F - 2.27}{1.82}\right)(E_{12} - E_{11})$

C3) $E_{CX1} = E_2 + \left(\frac{X}{4.75}\right)(E_{11} - E_2)$

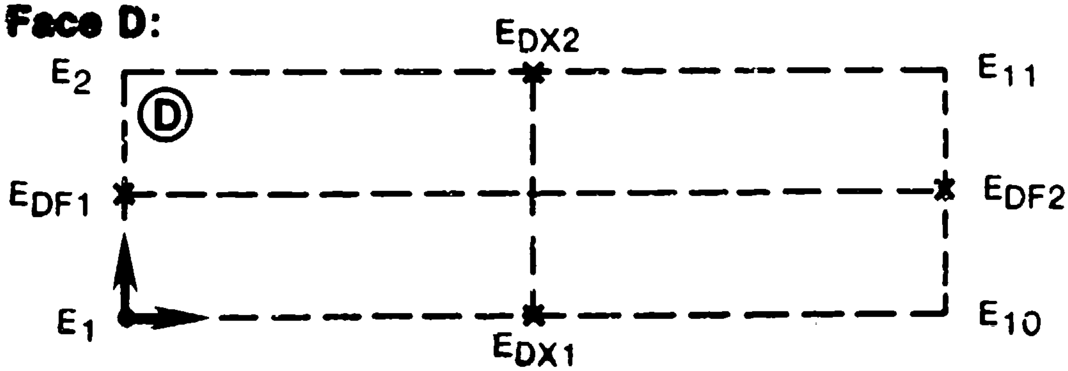
C4) $E_{CX2} = E_3 + \left(\frac{Y}{4.75}\right)(E_{12} - E_3)$

C5) $E_{CF} = E_{CF1} + \left(\frac{X}{4.75}\right)(E_{CF2} - E_{CF1})$

C6) $E_{CX} = E_{CX1} + \left(\frac{F - 2.27}{1.82}\right)(E_{CX2} - E_{CX1})$

C7) $E_C = \frac{E_{CF} + E_{CX}}{2}$

Face D:



D1) $E_{DF1} = E_1 + \left(\frac{F - 1.45}{0.82}\right)(E_2 - E_1)$

D2) $E_{DF2} = E_{10} + \left(\frac{F - 1.45}{0.82}\right)(E_{11} - E_{10})$

D3) $E_{DX1} = E_1 + \left(\frac{X}{4.75}\right)(E_{10} - E_1)$

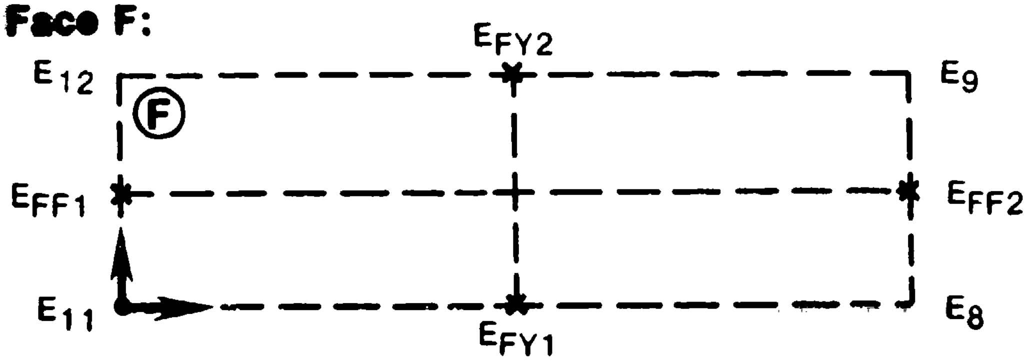
D4) $E_{DX2} = E_2 + \left(\frac{X}{4.75}\right)(E_{11} - E_2)$

D5) $E_{DF} = E_{DF1} + \left(\frac{X}{4.75}\right)(E_{DF2} - E_{DF1})$

D6) $E_{DX} = E_{DX1} + \left(\frac{F - 1.45}{0.82}\right)(E_{DX2} - E_{DX1})$

D7) $E_D = \frac{E_{DF} + E_{DX}}{2}$

Face F:



F1) $E_{FF1} = E_{11} + \left(\frac{F - 2.27}{1.82}\right)(E_{12} - E_{11})$

F2) $E_{FF2} = E_{8} + \left(\frac{F - 2.27}{1.82}\right)(E_{9} - E_{8})$

F3) $E_{FY1} = E_{11} + \left(\frac{Y}{2.25}\right)(E_{8} - E_{11})$

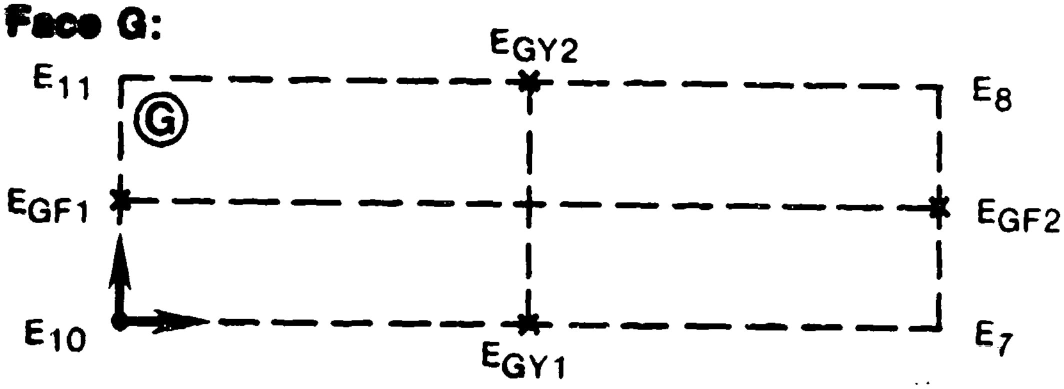
F4) $E_{FY2} = E_{12} + \left(\frac{Y}{2.25}\right)(E_{9} - E_{12})$

F5) $E_{FF} = E_{FF1} + \left(\frac{Y}{2.25}\right)(E_{FF2} - E_{FF1})$

F6) $E_{FY} = E_{FY1} + \left(\frac{F - 2.27}{1.82}\right)(E_{FY2} - E_{FY1})$

F7) $E_{F} = \frac{E_{FF} + E_{FY}}{2}$

Face G:



G1) $E_{GF1} = E_{10} + \left(\frac{F - 1.45}{0.82}\right)(E_{11} - E_{10})$

G2) $E_{GF2} = E_{7} + \left(\frac{F - 1.45}{0.82}\right)(E_{8} - E_{7})$

G3) $E_{GY1} = E_{1C} + \left(\frac{Y}{2.25}\right)(E_{7} - E_{10})$

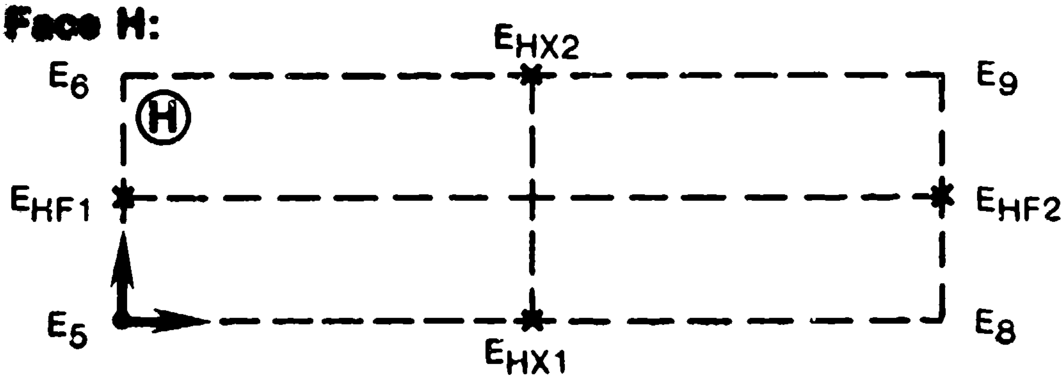
G4) $E_{GY2} = E_{11} + \left(\frac{Y}{2.25}\right)(E_{8} - E_{11})$

G5) $E_{GF} = E_{GF1} + \left(\frac{Y}{2.25}\right)(E_{GF2} - E_{GF1})$

G6) $E_{GY} = E_{GY1} + \left(\frac{F - 1.45}{0.82}\right)(E_{GY2} - E_{GY1})$

G7) $E_{G} = \frac{E_{GF} + E_{GY}}{2}$

Face H:



H1) $E_{HF1} = E_5 + \left(\frac{F - 2.27}{1.82}\right)(E_6 - E_5)$

H2) $E_{HF2} = E_8 + \left(\frac{F - 2.27}{1.82}\right)(E_9 - E_8)$

H3) $E_{HX1} = E_5 + \left(\frac{X}{4.75}\right)(E_8 - E_5)$

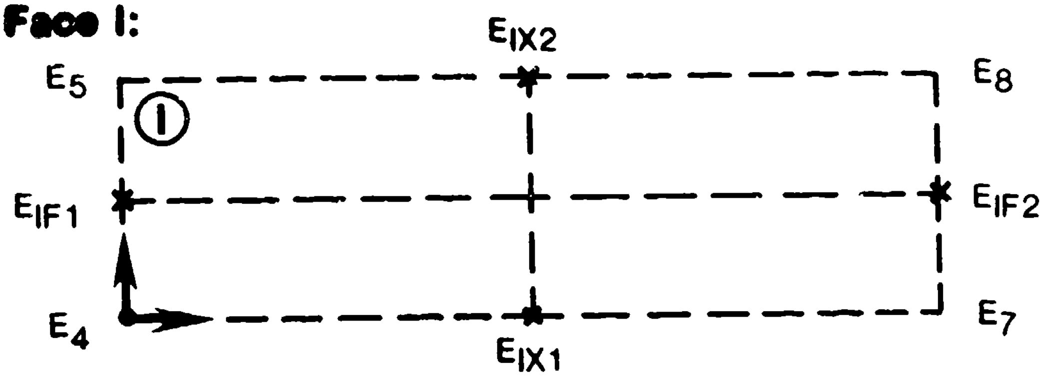
H4) $E_{HX2} = E_6 + \left(\frac{X}{4.75}\right)(E_9 - E_6)$

H5) $E_{HF} = E_{HF1} + \left(\frac{X}{4.75}\right)(E_{HF2} - E_{HF1})$

H6) $E_{HX} = E_{HX1} + \left(\frac{F - 2.27}{1.82}\right)(E_{HX2} - E_{HX1})$

H7) $E_H = \frac{E_{HF} + E_{HX}}{2}$

Face I:



I1) $E_{IF1} = E_4 + \left(\frac{F - 1.45}{0.82}\right)(E_5 - E_4)$

I2) $E_{IF2} = E_7 + \left(\frac{F - 1.45}{0.82}\right)(E_8 - E_7)$

I3) $E_{IX1} = E_4 + \left(\frac{X}{4.75}\right)(E_7 - E_4)$

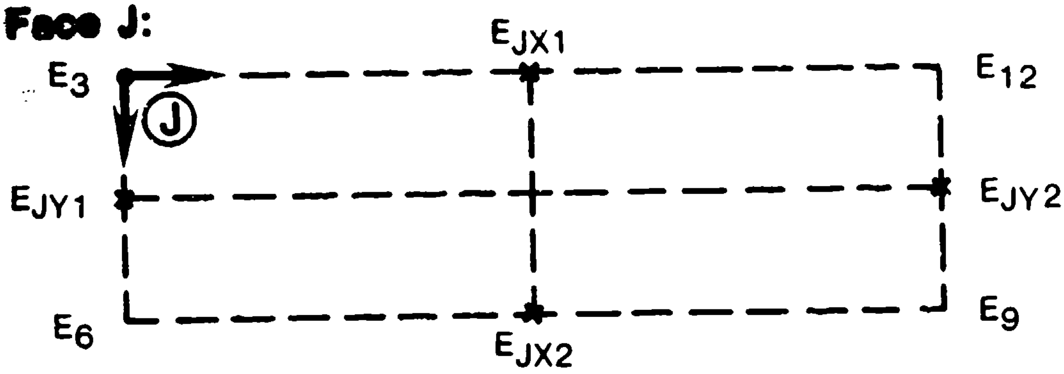
I4) $E_{IX2} = E_5 + \left(\frac{X}{4.75}\right)(E_8 - E_5)$

I5) $E_{IF} = E_{IF1} + \left(\frac{X}{4.75}\right)(E_{IF2} - E_{IF1})$

I6) $E_{IX} = E_{IX1} + \left(\frac{F - 1.45}{0.82}\right)(E_{IX2} - E_{IX1})$

I7) $E_I = \frac{E_{IF} + E_{IX}}{2}$

Face J:



J1) $E_{JY1} = E_3 + \left(\frac{Y}{2.25}\right) (E_6 - E_3)$

J2) $E_{JY2} = E_{12} + \left(\frac{Y}{2.25}\right) (E_9 - E_{12})$

J3) $E_{JX1} = E_3 + \left(\frac{X}{4.75}\right) (E_{12} - E_3)$

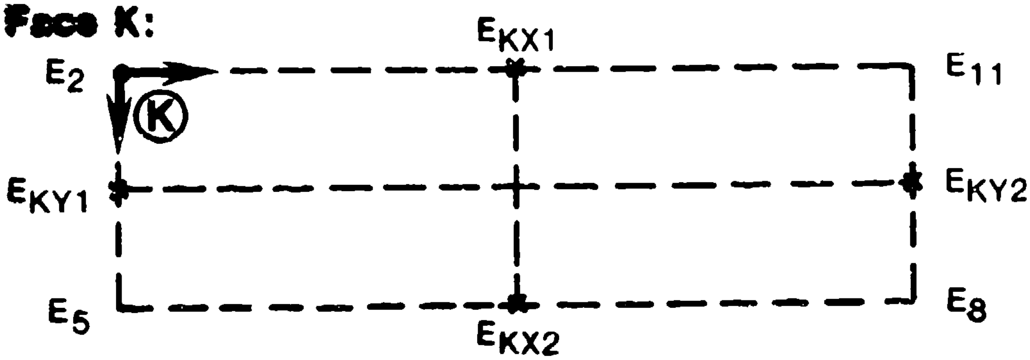
J4) $E_{JX2} = E_6 + \left(\frac{X}{4.75}\right) (E_9 - E_6)$

J5) $E_{JY} = E_{JY1} + \left(\frac{X}{4.75}\right) (E_{JY2} - E_{JY1})$

J6) $E_{JX} = E_{JX1} + \left(\frac{Y}{2.25}\right) (E_{JX2} - E_{JX1})$

J7) $E_J = \frac{E_{JY} + E_{JX}}{2}$

Face K:



K1) $E_{KY1} = E_2 + \left(\frac{Y}{2.25}\right)(E_5 - E_2)$

K2) $E_{KY2} = E_{11} + \left(\frac{Y}{2.25}\right)(E_8 - E_{11})$

K3) $E_{KX1} = E_2 + \left(\frac{X}{4.75}\right)(E_{11} - E_2)$

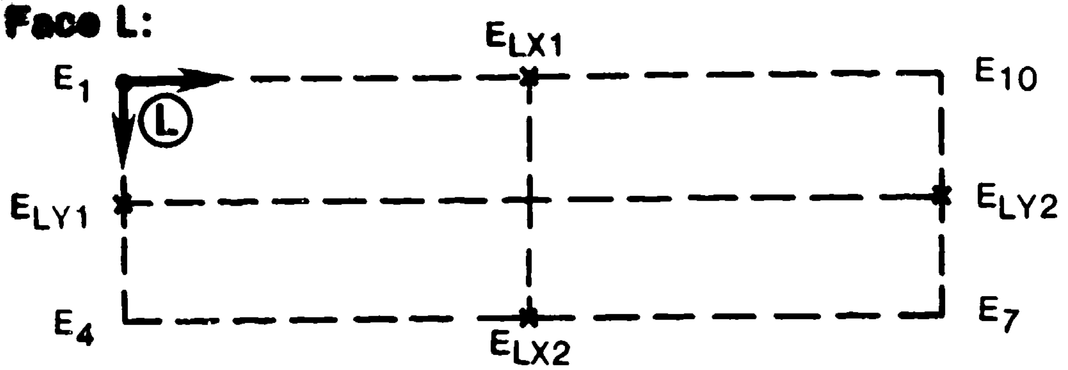
K4) $E_{KX2} = E_5 + \left(\frac{X}{4.75}\right)(E_8 - E_5)$

K5) $E_{KY} = E_{KY1} + \left(\frac{X}{4.75}\right)(E_{KY2} - E_{KY1})$

K6) $E_{KX} = E_{KX1} + \left(\frac{Y}{2.25}\right)(E_{KX2} - E_{KX1})$

K7) $E_K = \frac{E_{KY} + E_{KX}}{2}$

Face L:



L1) $E_{LY1} = E_1 + \left(\frac{Y}{2.25}\right)(E_4 - E_1)$

L2) $E_{LY2} = E_{10} + \left(\frac{Y}{2.25}\right)(E_7 - E_{10})$

L3) $E_{LX1} = E_1 + \left(\frac{X}{4.75}\right)(E_{10} - E_1)$

L4) $E_{LX2} = E_4 + \left(\frac{X}{4.75}\right)(E_7 - E_4)$

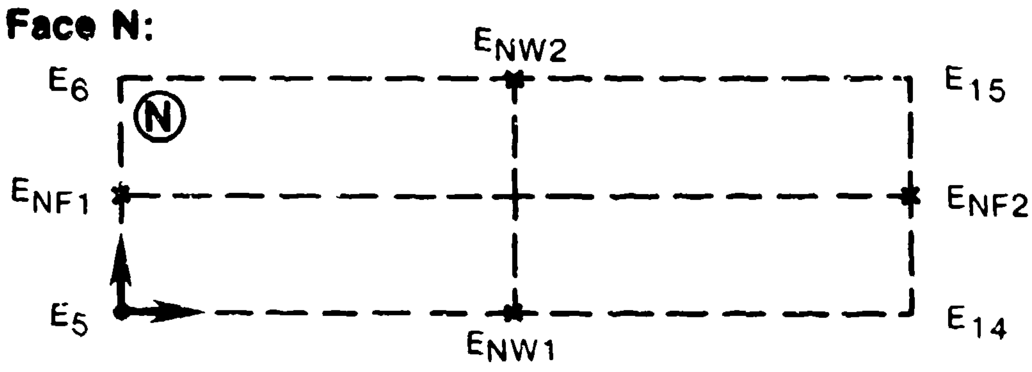
L5) $E_{LY} = E_{LY1} + \left(\frac{X}{4.75}\right)(E_{LY2} - E_{LY1})$

L6) $E_{LX} = E_{LX1} + \left(\frac{Y}{2.25}\right)(E_{LX2} - E_{LX1})$

L7) $E_L = \frac{E_{LY} + E_{LX}}{2}$

APPENDIX B

SUBSIDIARY EQUATIONS (FACE ELEMENTS N-V)



N1) $E_{NF1} = E_5 + \left(\frac{F - 2.27}{1.82}\right)(E_6 - E_5)$

N2) $E_{NF2} = E_{14} + \left(\frac{F - 2.27}{1.82}\right)(E_{15} - E_{14})$

N3) $E_{NW1} = E_5 + \left(\frac{W}{2.25}\right)(E_{14} - E_5)$

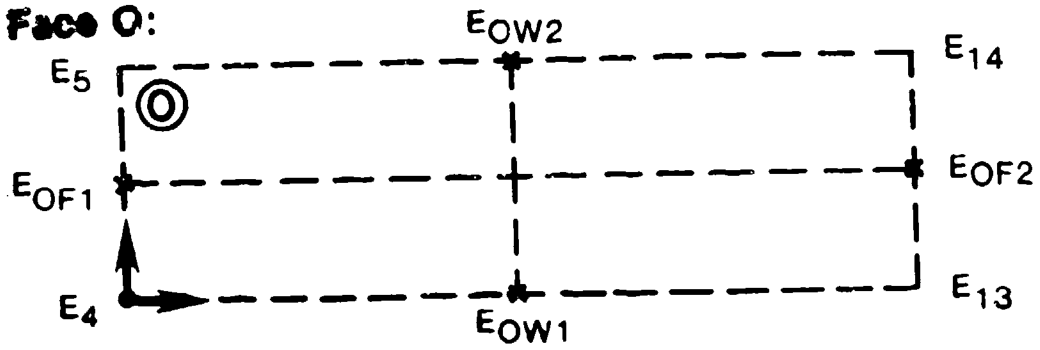
N4) $E_{NW2} = E_6 + \left(\frac{W}{2.25}\right)(E_{15} - E_6)$

N5) $E_{NF} = E_{NF1} + \left(\frac{W}{2.25}\right)(E_{NF2} - E_{NF1})$

N6) $E_{NW} = E_{NW1} + \left(\frac{F - 2.27}{1.82}\right)(E_{NW2} - E_{NW1})$

N7) $E_N = \frac{E_{NF} + E_{NW}}{2}$

Face O:



O1) $E_{OF1} = E_4 + \left(\frac{F - 1.45}{0.82}\right)(E_5 - E_4)$

O2) $E_{OF2} = E_{13} + \left(\frac{F - 1.45}{0.82}\right)(E_{14} - E_{13})$

O3) $E_{OW1} = E_4 + \left(\frac{W}{2.25}\right)(E_{13} - E_4)$

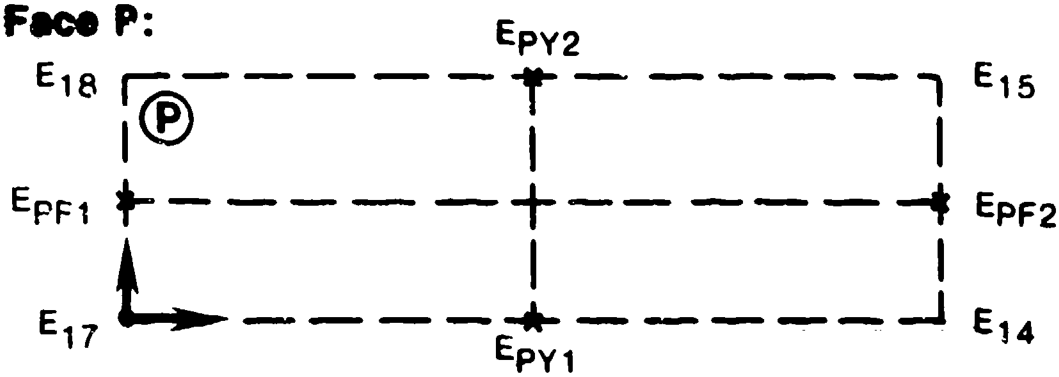
O4) $E_{OW2} = E_5 + \left(\frac{W}{2.25}\right)(E_{14} - E_5)$

O5) $E_{OF} = E_{OF1} + \left(\frac{W}{2.25}\right)(E_{OF2} - E_{OF1})$

O6) $E_{OW} = E_{OW1} + \left(\frac{F - 1.45}{0.82}\right)(E_{OW2} - E_{OW1})$

N7) $E_O = \frac{E_{OF} + E_{OW}}{2}$

Face P:



P1) $E_{PF1} = E_{17} + \left(\frac{F - 2.27}{1.82}\right)(E_{18} - E_{17})$

P2) $E_{PF2} = E_{14} + \left(\frac{F - 2.27}{1.82}\right)(E_{15} - E_{14})$

P3) $E_{PY1} = E_{17} + \left(\frac{Y}{2.25}\right)(E_{14} - E_{17})$

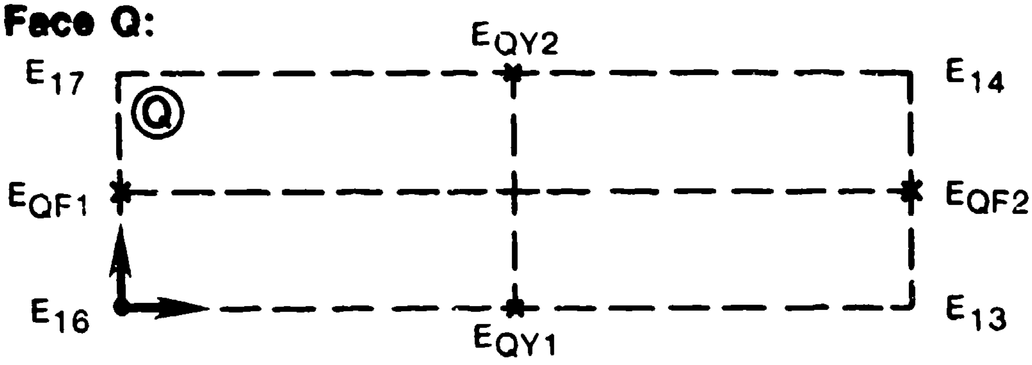
P4) $E_{PY2} = E_{18} + \left(\frac{Y}{2.25}\right)(E_{15} - E_{18})$

P5) $E_{PF} = E_{PF1} + \left(\frac{Y}{2.25}\right)(E_{PF2} - E_{PF1})$

P6) $E_{PY} = E_{PY1} + \left(\frac{F - 2.27}{1.82}\right)(E_{PY2} - E_{PY1})$

P7) $E_{P} = \frac{E_{PF} + E_{PY}}{2}$

Face Q:



Q1) $E_{QF1} = E_{16} + \left(\frac{F - 1.45}{0.82}\right)(E_{17} - E_{16})$

Q2) $E_{QF2} = E_{13} + \left(\frac{F - 1.45}{0.82}\right)(E_{14} - E_{13})$

Q3) $E_{QY1} = E_{16} + \left(\frac{Y}{2.25}\right)(E_{13} - E_{16})$

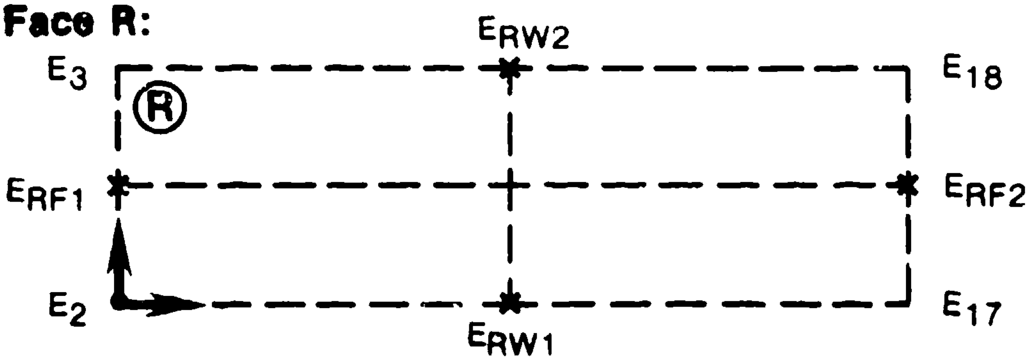
Q4) $E_{QY2} = E_{17} + \left(\frac{Y}{2.25}\right)(E_{14} - E_{17})$

Q5) $E_{QF} = E_{QF1} + \left(\frac{Y}{2.25}\right)(E_{QF2} - E_{QF1})$

Q6) $E_{QY} = E_{QY1} + \left(\frac{F - 1.45}{0.82}\right)(E_{QY2} - E_{QY1})$

Q7) $E_Q = \frac{E_{QF} + E_{QY}}{2}$

Face R:



R1) $E_{RF1} = E_2 + \left(\frac{F - 2.27}{1.82}\right)(E_3 - E_2)$

R2) $E_{RF2} = E_{17} + \left(\frac{F - 2.27}{1.82}\right)(E_{18} - E_{17})$

R3) $E_{RW1} = E_2 + \left(\frac{W}{2.25}\right)(E_{17} - E_2)$

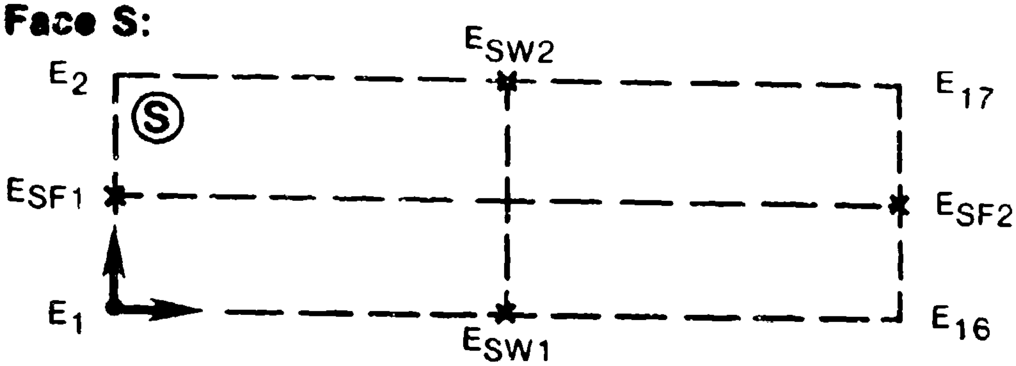
R4) $E_{RW2} = E_3 + \left(\frac{W}{2.25}\right)(E_{18} - E_3)$

R5) $E_{RF} = E_{RF1} + \left(\frac{W}{2.25}\right)(E_{RF2} - E_{RF1})$

R6) $E_{RW} = E_{RW1} + \left(\frac{F - 2.27}{1.82}\right)(E_{RW2} - E_{RW1})$

R7) $E_R = \frac{E_{RF} + E_{RW}}{2}$

Face S:



S1) $E_{SF1} = E_1 + \left(\frac{F - 1.45}{0.82}\right)(E_2 - E_1)$

S2) $E_{SF2} = E_{16} + \left(\frac{F - 1.45}{0.82}\right)(E_{17} - E_{16})$

S3) $E_{SW1} = E_1 + \left(\frac{W}{2.25}\right)(E_{16} - E_1)$

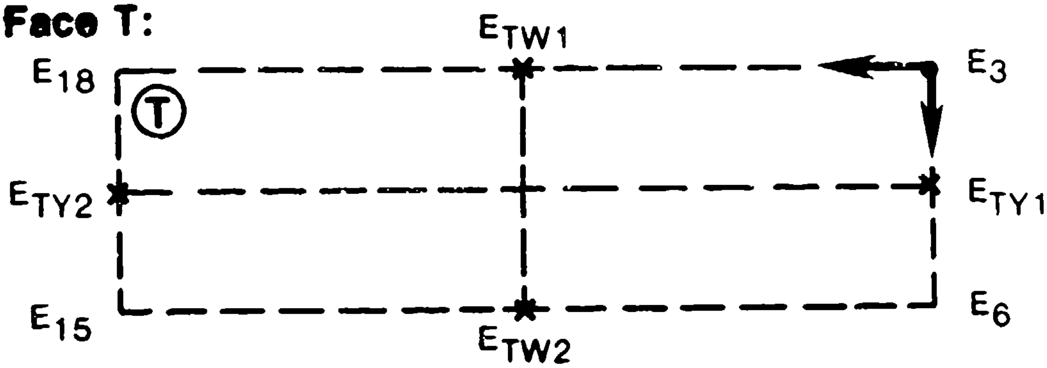
S4) $E_{SW2} = E_2 + \left(\frac{W}{2.25}\right)(E_{17} - E_2)$

S5) $E_{SF} = E_{SF1} + \left(\frac{W}{2.25}\right)(E_{SF2} - E_{SF1})$

S6) $E_{SW} = E_{SW1} + \left(\frac{F - 2.27}{1.82}\right)(E_{SW2} - E_{SW1})$

S7) $E_S = \frac{E_{SF} + E_{SW}}{2}$

Face T:



T1) $E_{TY1} = E_3 + \left(\frac{Y}{2.25}\right)(E_6 - E_3)$

T2) $E_{TY2} = E_{1'} \quad \frac{Y}{2.25}\Big)(E_{15} - E_{18})$

T3) $E_{TW1} = E_3 + \left(\frac{W}{2.25}\right)(E_{18} - E_3)$

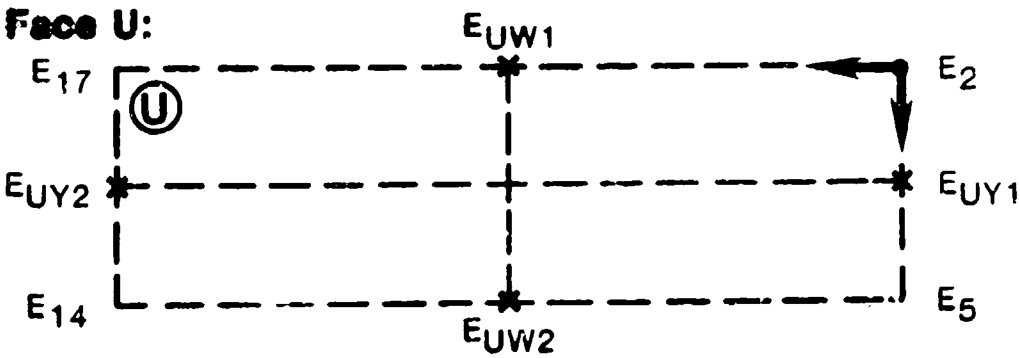
T4) $E_{TW2} = E_6 + \left(\frac{W}{2.25}\right)(E_{15} - E_6)$

T5) $E_{TY} = E_{TY1} + \left(\frac{W}{2.25}\right)(E_{TY2} - E_{TY1})$

T6) $E_{TW} = E_{TW1} + \left(\frac{Y}{2.25}\right)(E_{TW2} - E_{TW1})$

T7) $E_T = \frac{E_{TY} + E_{TF}}{2}$

Face U:



U1) $E_{UY1} = E_2 + (\frac{Y}{2.25})(E_5 - E_2)$

U2) $E_{UY2} = E_{17} + (\frac{Y}{2.25})(E_{14} - E_{17})$

U3) $E_{UW1} = E_2 + (\frac{W}{2.25})(E_{17} - E_2)$

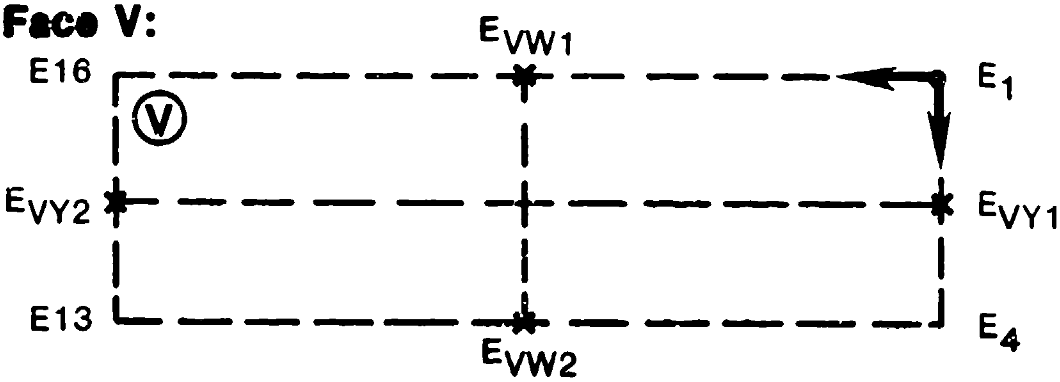
U4) $E_{UW2} = E_5 + (\frac{W}{2.25})(E_{14} - E_5)$

U5) $E_{UY} = E_{UY1} + (\frac{W}{2.25})(E_{UY2} - E_{UY1})$

U6) $E_{UW} = E_{UW1} + (\frac{Y}{2.25})(E_{UW2} - E_{UW1})$

U7) $E_U = \frac{E_{UY} + E_{UW}}{2}$

Face V:



V1) $E_{VY1} = E_1 + \left(\frac{Y}{2.25}\right)(E_4 - E_1)$

V2) $E_{VY2} = E_{16} + \left(\frac{Y}{2.25}\right)(E_{13} - E_{16})$

V3) $E_{VW1} = E_1 + \left(\frac{W}{2.25}\right)(E_{16} - E_1)$

V4) $E_{VW2} = E_4 + \left(\frac{W}{2.25}\right)(E_{13} - E_4)$

V5) $E_{VY} = E_{VY1} + \left(\frac{W}{2.25}\right)(E_{VY2} - E_{VY1})$

V6) $E_{VW} = E_{VW1} + \left(\frac{Y}{2.25}\right)(E_{VW2} - E_{VW1})$

V7) $E_V = \frac{E_{VY} + E_{VW}}{2}$

APPENDIX C

COMPUTER LISTING (MUSCLE FATIGUE PROGRAM)

```
25  HOME
30  PRINT "MATH MODEL OF NECK MUSCLE FATIGUE"
32  FOR PAUSE = 1 TO 1500: NEXT PAUSE
35  REM   I.E.D. & C.A.P. 6-FEB-84
50  REM  STORED CONSTANTS: ENDURANCE TIMES & STANDARD DEVIATIONS
60  REM   EA-ER=FWD.MEAN;XA-XR=LATERAL MEAN; YA-YR=FWD. STD. DEV.; ZA-ZR=L
     ATERAL STD. DEV..
70 EA = 98.25:EB = 74.40:EC = 67.60:ED = 75.40
75 EE = 65.60:EF = 95.80:EG = 48.80:EH = 37.40
80 EI = 41.40:EJ = 86.00:EK = 64.66:EL = 55.83
85 EM = 43.20:EN = 46.20:EO = 45.00:EP = 85.60
90 EQ = 64.20:ER = 83.25:XA = 106.33:XB = 83.00
95 XC = 86.00:XD = 155.80:XE = 81.00:XF = 71.60
100 XG = 105.20:XH = 102.20:XI = 122.20:XJ = 148.64
105 XK = 89.33:XL = 61.83:XM = 81.80:XN = 98.80
110 XO = 93.80:XP = 91.20:XQ = 73.40:XR = 128.60
115 YA = 34.297:YB = 43.166:YC = 33.975:YD = 56.849
120 YE = 20.948:YF = 61.280:YG = 18.674:YH = 9.915
125 YI = 8.877:YJ = 48.683:YK = 26.296:YL = 23.845
130 YM = 11.563:YN = 15.786:YO = 13.398:YP = 33.650
135 YQ = 19.435:YR = 20.320:ZA = 88.484:ZB = 33.106
140 ZC = 33.196:ZD = 58.896:ZE = 40.125:ZF = 39.633
145 ZG = 24.813:ZH = 26.414:ZI = 48.669:ZJ = 52.564
150 ZK = 42.321:ZL = 35.471:ZM = 30.777:ZN = 42.214
155 ZO = 30.793:ZP = 37.785:ZQ = 46.646:ZR = 69.013
159  HOME
160  INPUT "ENTER HELMET WEIGHT FROM 1.45 TO 4.09 KG. ";F: PRINT
170  INPUT "ENTER DISPLACEMENT ON X-AXIS FROM -2.25 CM. TO 4.75 CM. ";X: PRINT

180  INPUT "ENTER DISPLACEMENT ON Y-AXIS FROM -2.25 CM. TO 2.25 CM. ";YT: PRINT

190 Y =  ABS (YT)
195  REM  DATA OUT-OF-BOUNDS?
200  IF F < 1.45 GOTO 4000
```

```
210  IF F > 4.09 GOTO 4000
220  IF Y > 2.25 GOTO 4000
230  IF X > 4.75 GOTO 4000
240  IF X <  - 2.25 GOTO 4000
245  REM  DATA ROUTING
250  IF F < 2.27 GOTO 650
260  IF X < 0 GOTO 1040
265  REM  EQUATIONS FOR QUADRANT A
270 A1 = EB + (((F - 2.27) / 1.82) * (EC - EB))
272 A2 = EE + (((F - 2.27) / 1.82) * (EF - EE))
274 A3 = EB + ((Y / 2.25) * (EE - EB))
276 A4 = EC + ((Y / 2.25) * (EF - EC))
278 A5 = A1 + ((Y / 2.25) * (A2 - A1))
280 A6 = A3 + (((F - 2.27) / 1.82) * (A4 - A3))
282 A7 = ((A5 + A6) / 2)
284 C1 = EB + (((F - 2.27) / 1.82) * (EC - EB))
286 C2 = EK + (((F - 2.27) / 1.82) * (EL - EK))
288 C3 = EB + ((X / 4.75) * (EK - EB))
290 C4 = EC + ((X / 4.75) * (EL - EC))
292 C5 = C1 + ((X / 4.75) * (C2 - C1))
294 C6 = C3 + (((F - 2.27) / 1.82) * (C4 - C3))
296 C7 = ((C5 + C6) / 2)
298 F1 = EK + (((F - 2.27) / 1.82) * (EL - EK))
300 F2 = EH + (((F - 2.27) / 1.82) * (EI - EH))
302 F3 = EK + ((Y / 2.25) * (EH - EK))
304 F4 = EL + ((Y / 2.25) * (EI - EL))
306 F5 = F1 + ((Y / 2.25) * (F2 - F1))
308 F6 = F3 + (((F - 2.27) / 1.82) * (F4 - F3))
310 F7 = ((F5 + F6) / 2)
312 H1 = EE + (((F - 2.27) / 1.82) * (EF - EE))
314 H2 = EH + (((F - 2.27) / 1.82) * (EI - EH))
316 H3 = EE + ((X / 4.75) * (EH - EE))
318 H4 = EF + ((X / 4.75) * (EI - EF))
320 H5 = H1 + ((X / 4.75) * (H2 - H1))
322 H6 = H3 + (((F - 2.27) / 1.82) * (H4 - H3))
324 H7 = ((H5 + H6) / 2)
326 J1 = EC + ((Y / 2.25) * (EF - EC))
328 J2 = EL + ((Y / 2.25) * (EI - EL))
330 J3 = EC + ((X / 4.75) * (EL - EC))
332 J4 = EF + ((X / 4.75) * (EI - EF))
334 J5 = J1 + ((X / 4.75) * (J2 - J1))
336 J6 = J3 + ((Y / 2.25) * (J4 - J3))
338 J7 = ((J5 + J6) / 2)
340 K1 = EB + ((Y / 2.25) * (EE - EB))
342 K2 = EK + ((Y / 2.25) * (EH - EK))
344 K3 = EB + ((X / 4.75) * (EK - EB))
346 K4 = EE + ((X / 4.75) * (EH - EE))
348 K5 = K1 + ((X / 4.75) * (K2 - K1))
350 K6 = K3 + ((Y / 2.25) * (K4 - K3))
352 K7 = ((K5 + K6) / 2)
354 M1 = A7 + ((X / 4.75) * (F7 - A7))
356 M2 = C7 + ((Y / 2.25) * (H7 - C7))
358 M3 = K7 + (((F - 2.27) / 1.82) * (J7 - K7))
360 M4 = ((M1 + M2 + M3) / 3)
362 AA = XB + (((F - 2.27) / 1.82) * (XC - XB))
364 AB = XE + (((F - 2.27) / 1.82) * (XF - XE))
366 AC = XB + ((Y / 2.25) * (XE - XB))
368 AD = XC + ((Y / 2.25) * (XF - XC))
370 AE = AA + ((Y / 2.25) * (AB - AA))
372 AF = AC + (((F - 2.27) / 1.82) * (AD - AC))
374 AG = ((AE + AF) / 2)
376 CA = XB + (((F - 2.27) / 1.82) * (XC - XB))
378 CB = XK + (((F - 2.27) / 1.82) * (XL - XK))
380 CC = XB + ((X / 4.75) * (XK - XB))
382 CD = XC + ((X / 4.75) * (XL - XC))
384 CE = CA + ((X / 4.75) * (CB - CA))
```

```
384 CF = CC + (((F - 2.27) / 1.82) * (CD - CC))
388 CG = ((CE + CF) / 2)
390 FA = XK + (((F - 2.27) / 1.82) * (XL - XK))
392 FB = XH + (((F - 2.27) / 1.82) * (XI - XH))
394 FC = XK + ((Y / 2.25) * (XH - XK))
396 FD = XL + ((Y / 2.25) * (XI - XL))
398 FE = FA + ((Y / 2.25) * (FB - FA))
400 FF = FC + (((F - 2.27) / 1.82) * (FD - FC))
402 FG = ((FE + FF) / 2)
404 HA = XE + (((F - 2.27) / 1.82) * (XF - XE))
406 HB = XH + (((F - 2.27) / 1.82) * (XI - XH))
408 HC = XE + ((X / 4.75) * (XH - XE))
410 HD = XF + ((X / 4.75) * (XI - XF))
412 HE = HA + ((X / 4.75) * (HB - HA))
414 HF = HC + (((F - 2.27) / 1.82) * (HD - HC))
416 HG = ((HE + HF) / 2)
418 JA = XC + ((Y / 2.25) * (XF - XC))
420 JB = XL + ((Y / 2.25) * (XI - XL))
422 JC = XC + ((X / 4.75) * (XL - XC))
424 JD = XF + ((X / 4.75) * (XI - XF))
426 JE = JA + ((X / 4.75) * (JB - JA))
428 JF = JC + ((Y / 2.25) * (JD - JC))
430 JG = ((JE + JF) / 2)
432 KA = XB + ((Y / 2.25) * (XE - XB))
434 KB = XK + ((Y / 2.25) * (XH - XK))
436 KC = XB + ((X / 4.75) * (XK - XB))
438 KD = XE + ((X / 4.75) * (XH - XE))
440 KE = KA + ((X / 4.75) * (KB - KA))
442 KF = KC + ((Y / 2.25) * (KD - KC))
444 KG = ((KE + KF) / 2)
446 MA = AG + ((X / 4.75) * (FG - AG))
448 MB = CG + ((Y / 2.25) * (HG - CG))
450 MC = KG + (((F - 2.27) / 1.82) * (JG - KG))
452 MD = ((MA + MB + MC) / 3)
454 AH = YB + (((F - 2.27) / 1.82) * (YC - YD))
456 AI = YE + (((F - 2.27) / 1.82) * (YF - YE))
458 AJ = YB + ((Y / 2.25) * (YE - YB))
460 AK = YC + ((Y / 2.25) * (YF - YC))
462 AL = AH + ((Y / 2.25) * (AI - AH))
464 AM = AJ + (((F - 2.27) / 1.82) * (AK - AJ))
466 AN = ((AL + AM) / 2)
468 CH = YB + (((F - 2.27) / 1.82) * (YC - YB))
470 CI = YK + (((F - 2.27) / 1.82) * (YL - YK))
472 CJ = YB + ((X / 4.75) * (YK - YB))
474 CK = YC + ((X / 4.75) * (YL - YC))
476 CL = CH + ((X / 4.75) * (CI - CH))
478 CM = CJ + (((F - 2.27) / 1.82) * (CK - CJ))
480 CN = ((CL + CM) / 2)
482 FH = YK + (((F - 2.27) / 1.82) * (YL - YK))
484 FI = YH + (((F - 2.27) / 1.82) * (YI - YH))
486 FJ = YK + ((Y / 2.25) * (YH - YK))
488 FK = YL + ((Y / 2.25) * (YI - YL))
490 FL = FH + ((Y / 2.25) * (FI - FH))
492 FM = FJ + (((F - 2.27) / 1.82) * (FK - FJ))
494 FX = ((FL + FM) / 2)
496 HH = YE + (((F - 2.27) / 1.82) * (YF - YE))
498 HI = YH + (((F - 2.27) / 1.82) * (YI - YH))
500 HJ = YE + ((X / 4.75) * (YH - YE))
502 HK = YF + ((X / 4.75) * (YI - YF))
504 HL = HH + ((X / 4.75) * (HI - HH))
506 HM = HJ + (((F - 2.27) / 1.82) * (HK - HJ))
508 HN = ((HL + HM) / 2)
510 JH = YC + ((Y / 2.25) * (YF - YC))
512 JI = YL + ((Y / 2.25) * (YI - YL))
514 JJ = YC + ((X / 4.75) * (YL - YC))
516 JK = YF + ((X / 4.75) * (YI - YF))
```

```
518 JL = JH + ((X / 4.75) * (JI - JH))
520 JM = JJ + ((Y / 2.25) * (JK - JJ))
522 JN = ((JL + JM) / 2)
524 KH = YB + ((Y / 2.25) * (YE - YB))
526 KI = YK + ((Y / 2.25) * (YH - YK))
528 KJ = YB + ((X / 4.75) * (YK - YB))
530 KK = YE + ((X / 4.75) * (YH - YE))
532 KL = KH + ((X / 4.75) * (KI - KH))
534 KM = KJ + ((Y / 2.25) * (KK - KJ))
536 KN = ((KL + KM) / 2)
538 MI = AN + ((X / 4.75) * (FX - AN))
540 MJ = CN + ((Y / 2.25) * (HN - CN))
542 MK = KN + (((F - 2.27) / 1.82) * (JN - KN))
544 ML = ((MI + MJ + MK) / 3)
546 AO = ZB + (((F - 2.27) / 1.82) * (ZC - ZB))
548 AP = ZE + (((F - 2.27) / 1.82) * (ZF - ZE))
550 AQ = ZB + ((Y / 2.25) * (ZE - ZB))
552 AR = ZC + ((Y / 2.25) * (ZF - ZC))
554 AS = AO + ((Y / 2.25) * (AP - AO))
556 AX = AQ + (((F - 2.27) / 1.82) * (AR - AQ))
558 AU = ((AS + AX) / 2)
560 CO = ZB + (((F - 2.27) / 1.82) * (ZC - ZB))
562 CP = ZK + (((F - 2.27) / 1.82) * (ZL - ZK))
564 CQ = ZB + ((X / 4.75) * (ZK - ZB))
566 CR = ZC + ((X / 4.75) * (ZL - ZC))
568 CS = CO + ((X / 4.75) * (CP - CO))
570 CT = CQ + (((F - 2.27) / 1.82) * (CR - CQ))
572 CU = ((CS + CT) / 2)
574 FO = ZK + (((F - 2.27) / 1.82) * (ZL - ZK))
576 FP = ZH + (((F - 2.27) / 1.82) * (ZI - ZH))
578 FQ = ZK + ((Y / 2.25) * (ZH - ZK))
580 FR = ZL + ((Y / 2.25) * (ZI - ZL))
582 FS = FO + ((Y / 2.25) * (FP - FO))
584 FT = FQ + (((F - 2.27) / 1.82) * (FR - FQ))
586 FU = ((FS + FT) / 2)
588 HO = ZE + (((F - 2.27) / 1.82) * (ZF - ZE))
590 HP = ZH + (((F - 2.27) / 1.82) * (ZI - ZH))
592 HQ = ZE + ((X / 4.75) * (ZH - ZE))
594 HR = ZF + ((X / 4.75) * (ZI - ZF))
596 HS = HO + ((X / 4.75) * (HP - HO))
598 HT = HQ + (((F - 2.27) / 1.82) * (HR - HQ))
600 HU = ((HS + HT) / 2)
602 JO = ZC + ((Y / 2.25) * (ZF - ZC))
604 JP = ZL + ((Y / 2.25) * (ZI - ZL))
606 JQ = ZC + ((X / 4.75) * (ZL - ZC))
608 JR = ZF + ((X / 4.75) * (ZI - ZF))
610 JS = JO + ((X / 4.75) * (JP - JO))
612 JT = JQ + ((Y / 2.25) * (JR - JQ))
614 JU = ((JS + JT) / 2)
616 KO = ZB + ((Y / 2.25) * (ZE - ZB))
618 KP = ZK + ((Y / 2.25) * (ZH - ZK))
620 KQ = ZB + ((X / 4.75) * (ZK - ZB))
622 KR = ZE + ((X / 4.75) * (ZH - ZE))
624 KS = KO + ((X / 4.75) * (KP - KO))
626 KT = KQ + ((Y / 2.25) * (KR - KQ))
628 KU = ((KS + KT) / 2)
630 MQ = AU + ((X / 4.75) * (FU - AU))
632 MR = CU + ((Y / 2.25) * (HU - CU))
634 MS = KU + (((F - 2.27) / 1.82) * (JU - KU))
636 MT = ((MQ + MR + MS) / 3)
640  GOSUB 1835
642  INPUT "DO YOU WANT A HARCOPY? ENTER Y(YES) OR N(NO) ";A$
643  IF A$ = "N" GOTO 4005
644  IF A$ = "Y" THEN  PR# 1: GOSUB 1840
645  PR# 0
646  GOTO 4005
```

```
650  IF X < 0 GOTO 1430
655  REM  EQUATIONS FOR QUADRANT B
660 B1 = EA + (((F - 1.45) / .82) * (EB - EA))
662 B2 = ED + (((F - 1.45) / .82) * (EE - ED))
664 B3 = EA + ((Y / 2.25) * (ED - EA))
666 B4 = EB + ((Y / 2.25) * (EE - EB))
668 B5 = B1 + ((Y / 2.25) * (B2 - B1))
670 B6 = B3 + (((F - 1.45) / .82) * (B4 - B3))
672 B7 = ((B5 + B6) / 2)
674 D1 = EA + (((F - 1.45) / .82) * (EB - EA))
676 D2 = EJ + (((F - 1.45) / .82) * (EK - EJ))
678 D3 = EA + ((X / 4.75) * (EJ - EA))
680 D4 = EB + ((X / 4.75) * (EK - EB))
682 D5 = D1 + ((X / 4.75) * (D2 - D1))
684 D6 = D3 + (((F - 1.45) / .82) * (D4 - D3))
686 D7 = ((D5 + D6) / 2)
688 G1 = EJ + (((F - 1.45) / .82) * (EK - EJ))
690 G2 = EG + (((F - 1.45) / .82) * (EH - EG))
692 G3 = EJ + ((Y / 2.25) * (EG - EJ))
694 G4 = EK + ((Y / 2.25) * (EH - EK))
696 G5 = G1 + ((Y / 2.25) * (G2 - G1))
698 G6 = G3 + (((F - 1.45) / .82) * (G4 - G3))
700 G7 = ((G5 + G6) / 2)
702 I1 = ED + (((F - 1.45) / .82) * (EE - ED))
704 I2 = EG + (((F - 1.45) / .82) * (EH - EG))
706 I3 = ED + ((X / 4.75) * (EG - ED))
708 I4 = EE + ((X / 4.75) * (EH - EE))
710 I5 = I1 + ((X / 4.75) * (I2 - I1))
712 I6 = I3 + (((F - 1.45) / .82) * (I4 - I3))
714 I7 = ((I5 + I6) / 2)
716 K1 = EB + ((Y / 2.25) * (EE - EB))
718 K2 = EK + ((Y / 2.25) * (EH - EK))
720 K3 = EB + ((X / 4.75) * (EK - EB))
722 K4 = EE + ((X / 4.75) * (EH - EE))
724 K5 = K1 + ((X / 4.75) * (K2 - K1))
726 K6 = K3 + ((Y / 2.25) * (K4 - K3))
728 K7 = ((K5 + K6) / 2)
730 L1 = EA + ((Y / 2.25) * (ED - EA))
732 L2 = EJ + ((Y / 2.25) * (EG - EJ))
734 L3 = EA + ((X / 4.75) * (EJ - EA))
736 L4 = ED + ((X / 4.75) * (EG - ED))
738 L5 = L1 + ((X / 4.75) * (L2 - L1))
740 L6 = L3 + ((Y / 2.25) * (L4 - L3))
742 L7 = ((L5 + L6) / 2)
744 M5 = B7 + ((X / 4.75) * (G7 - B7))
746 M6 = D7 + ((Y / 2.25) * (I7 - D7))
748 M7 = L7 + (((F - 1.45) / .82) * (K7 - L7))
750 M8 = ((M5 + M6 + M7) / 3)
752 BA = XA + (((F - 1.45) / .82) * (XB - XA))
754 BB = XD + (((F - 1.45) / .82) * (XE - XD))
756 BC = XA + ((Y / 2.25) * (XD - XA))
758 BD = XB + ((Y / 2.25) * (XE - XB))
760 BE = BA + ((Y / 2.25) * (BB - BA))
762 BF = BC + (((F - 1.45) / .82) * (BD - BC))
764 BG = ((BE + BF) / 2)
766 DA = XA + (((F - 1.45) / .82) * (XB - XA))
768 DB = XJ + (((F - 1.45) / .82) * (XK - XJ))
770 DC = XA + ((X / 4.75) * (XJ - XA))
772 DD = XB + ((X / 4.75) * (XK - XB))
774 DE = DA + ((X / 4.75) * (DB - DA))
776 DF = DC + (((F - 1.45) / .82) * (DD - DC))
778 DG = ((DE + DF) / 2)
780 GA = XJ + (((F - 1.45) / .82) * (XK - XJ))
782 GB = XG + (((F - 1.45) / .82) * (XH - XG))
784 GC = XJ + ((Y / 2.25) * (XG - XJ))
786 GD = XK + ((Y / 2.25) * (XH - XK))
```

```
788 GE = GA + ((Y / 2.25) * (GB - GA))
790 GF = GC + (((F - 1.45) / .82) * (GD - GC))
792 GG = ((GE + GF) / 2)
794 IA = XD + (((F - 1.45) / .82) * (XE - XD))
796 IB = XG + (((F - 1.45) / .82) * (XH - XG))
798 IC = XD + ((X / 4.75) * (XG - XD))
800 ID = XE + ((X / 4.75) * (XH - XE))
802 IE = IA + ((Y / 4.75) * (IB - IA))
804 IX = IC + (((F - 1.45) / .82) * (ID - IC))
806 IG = ((IE + IX) / 2)
808 KA = XB + ((Y / 2.25) * (XE - XB))
810 KB = XK + ((Y / 2.25) * (XH - XK))
812 KC = XB + ((X / 4.75) * (XK - XB))
814 KD = XE + ((X / 4.75) * (XH - XE))
816 KE = KA + ((X / 4.75) * (KB - KA))
818 KF = KC + ((Y / 2.25) * (KD - KC))
820 KG = ((KE + KF) / 2)
822 LA = XA + ((Y / 2.25) * (XD - XA))
824 LB = XJ + ((Y / 2.25) * (XG - XJ))
826 LC = XA + ((X / 4.75) * (XJ - XA))
828 LD = XD + ((X / 4.75) * (XG - XD))
830 LE = LA + ((X / 4.75) * (LB - LA))
832 LF = LC + ((Y / 2.25) * (LD - LC))
834 LG = ((LE + LF) / 2)
836 ME = BG + ((X / 4.73) * (GG - BG))
838 MF = DG + ((Y / 2.25) * (IG - DG))
840 MG = LG + (((F - 1.45) / .82) * (KG - LG))
842 MH = ((ME + MF + MG) / 3)
844 BH = YA + (((F - 1.45) / .82) * (YB - YA))
846 BI = YD + (((F - 1.45) / .82) * (YE - YD))
848 BJ = YA + ((Y / 2.25) * (YD - YA))
850 BK = YB + ((Y / 2.25) * (YE - YB))
852 BL = BH + ((Y / 2.25) * (BI - BH))
854 BM = BJ + (((F - 1.45) / .82) * (BK - BJ))
856 BN = ((BL + BM) / 2)
858 DH = YA + (((F - 1.45) / .82) * (YB - YA))
860 DI = YJ + (((F - 1.45) / .82) * (YK - YJ))
862 DJ = YA + ((X / 4.75) * (YJ - YA))
864 DK = YB + ((X / 4.75) * (YK - YB))
866 DL = DH + ((X / 4.75) * (DI - DH))
868 DM = DJ + (((F - 1.45) / .82) * (DK - DJ))
870 DN = ((DL + DM) / 2)
872 GH = YJ + (((F - 1.45) / .82) * (YK - YJ))
874 GI = YG + (((F - 1.45) / .82) * (YH - YG))
876 GJ = YJ + ((Y / 2.25) * (YG - YJ))
878 GK = YK + ((Y / 2.25) * (YH - YK))
880 GL = GH + ((Y / 2.25) * (GI - GH))
882 GM = GJ + (((F - 1.45) / .82) * (GK - GJ))
884 GN = ((GL + GM) / 2)
886 IH = YD + (((F - 1.45) / .82) * (YE - YD))
888 II = YG + (((F - 1.45) / .82) * (YH - YG))
890 IJ = YD + ((X / 4.75) * (YG - YD))
892 IK = YE + ((X / 4.75) * (YH - YE))
894 IL = IH + ((X / 4.75) * (II - IH))
896 IM = IJ + (((F - 1.45) / .82) * (IK - IJ))
898 IN = ((IL + IM) / 2)
900 KH = YB + ((Y / 2.25) * (YE - YB))
902 KI = YK + ((Y / 2.25) * (YH - YK))
904 KJ = YB + ((X / 4.75) * (YK - YB))
906 KK = YE + ((X / 4.75) * (YH - YE))
908 KL = KH + ((X / 4.75) * (KI - KH))
910 KM = KJ + ((Y / 2.25) * (KK - KJ))
912 KN = ((KL + KM) / 2)
914 LH = YA + ((Y / 2.25) * (YD - YA))
916 LI = YJ + ((Y / 2.25) * (YG - YJ))
918 LJ = YA + ((X / 4.75) * (YJ - YA))
```

```
920 LK = YD + ((X / 4.75) * (YG - YD))
922 LL = LH + ((X / 4.75) * (LI - LH))
924 LM = LJ + ((Y / 2.25) * (LK - LJ))
926 LN = ((LL + LM) / 2)
928 MM = BN + ((X / 4.75) * (GN - BN))
930 MN = DN + ((Y / 2.25) * (IN - DN))
932 MO = LN + (((F - 1.45) / .82) * (KN - LN))
934 MP = ((MM + MN + MO) / 3)
936 BO = ZA + (((F - 1.45) / .82) * (ZB - ZA))
938 BP = ZD + (((F - 1.45) / .82) * (ZE - ZD))
940 BQ = ZA + ((Y / 2.25) * (ZD - ZA))
942 BR = ZB + ((Y / 2.25) * (ZE - ZB))
944 BS = BO + ((Y / 2.25) * (BP - BO))
946 BT = BQ + (((F - 1.45) / .82) * (BR - BQ))
948 BU = ((BS + BT) / 2)
950 DO = ZA + (((F - 1.45) / .82) * (ZB - ZA))
952 DP = ZJ + (((F - 1.45) / .82) * (ZK - ZJ))
954 DQ = ZA + ((X / 4.75) * (ZJ - ZA))
956 DR = ZB + ((X / 4.75) * (ZK - ZB))
958 DS = DO + ((X / 4.75) * (DP - DO))
960 DT = DQ + (((F - 1.45) / .82) * (DR - DQ))
962 DU = ((DS + DT) / 2)
964 GO = ZJ + (((F - 1.45) / .82) * (ZK - ZJ))
966 GP = ZG + (((F - 1.45) / .82) * (ZH - ZG))
968 GQ = ZJ + ((Y / 2.25) * (ZG - ZJ))
970 GX = ZK + ((Y / 2.25) * (ZH - ZK))
972 GS = GO + ((Y / 2.25) * (GP - GO))
974 GT = GQ + (((F - 1.45) / .82) * (GX - GQ))
976 GU = ((GS + GT) / 2)
978 IO = ZD + (((F - 1.45) / .82) * (ZE - ZD))
980 IP = ZG + (((F - 1.45) / .82) * (ZH - ZG))
982 IQ = ZD + ((X / 4.75) * (ZG - ZD))
984 IR = ZE + ((X / 4.75) * (ZH - ZE))
986 IS = IO + ((X / 4.75) * (IP - IO))
988 IT = IQ + (((F - 1.45) / .82) * (IR - IQ))
990 IU = ((IS + IT) / 2)
992 KO = ZB + ((Y / 2.25) * (ZE - ZB))
994 XP = ZK + ((Y / 2.25) * (ZH - ZK))
996 KQ = ZB + ((X / 4.75) * (ZK - ZB))
998 KR = ZE + ((X / 4.75) * (ZH - ZE))
1000 KS = KO + ((X / 4.75) * (KP - KO))
1002 KT = KQ + ((Y / 2.25) * (KR - KQ))
1004 KU = ((KS + KT) / 2)
1006 LO = ZA + ((Y / 2.25) * (ZD - ZA))
1008 LP = ZJ + ((Y / 2.25) * (ZG - ZJ))
1010 LQ = ZA + ((X / 4.75) * (ZJ - ZA))
1012 LR = ZD + ((X / 4.75) * (ZG - ZD))
1014 LS = LO + ((X / 4.75) * (LP - LO))
1016 LT = LQ + ((Y / 2.25) * (LR - LQ))
1018 LU = ((LS + LT) / 2)
1020 MU = BU + ((X / 4.75) * (GU - BU))
1022 MV = DU + ((Y / 2.25) * (IU - DU))
1024 MW = LU + (((F - 1.45) / .82) * (KU - LU))
1026 MX = ((MU + MV + MW) / 3)
1030  GOSUB 1895
1032  INPUT "DO YOU WANT A HARDCOPY? ENTER Y(YES) OR N(NO) ";A$
1033  IF A$ = "N" GOTO 4005
1034  IF A$ = "Y" THEN  PR# 1: GOSUB 1900
1035  PR# 0
1036  GOTO 4005
1040 W =  - X
1045  REM  EQUATIONS FOR QUADRANT C
1050 A1 = EB + (((F - 2.27) / 1.82) * (EC - EB))
1052 A2 = EE + (((F - 2.27) / 1.82) * (EF - EE))
1054 A3 = EB + ((Y / 2.25) * (EE - EB))
1056 A4 = EC + ((Y / 2.25) * (EF - EC))
```

```
1058 A5 = A1 + ((Y / 2.25) * (A2 - A1))
1060 A6 = A3 + (((F - 2.27) / 1.82) * (A4 - A3))
1062 A7 = ((A5 + A6) / 2)
1064 N1 = EE + (((F - 2.27) / 1.82) * (EF - EE))
1066 N2 = EN + (((F - 2.27) / 1.82) * (EO - EN))
1068 N3 = EE + ((W / 2.25) * (EN - EE))
1070 N4 = EF + ((W / 2.25) * (EO - EF))
1072 N5 = N1 + ((W / 2.25) * (N2 - N1))
1074 N6 = N3 + (((F - 2.27) / 1.82) * (N4 - N3))
1076 N7 = ((N5 + N6) / 2)
1078 P1 = EQ + (((F - 2.27) / 1.82) * (ER - EQ))
1080 P2 = EN + (((F - 2.27) / 1.82) * (EO - EN))
1082 P3 = EQ + ((Y / 2.25) * (EN - EQ))
1084 P4 = ER + ((Y / 2.25) * (EO - ER))
1086 P5 = P1 + ((Y / 2.25) * (P2 - P1))
1088 P6 = P3 + (((F - 2.27) / 1.82) * (P4 - P3))
1090 P7 = ((P5 + P6) / 2)
1092 R1 = EB + (((F - 2.27) / 1.82) * (EC - EB))
1094 R2 = EQ + (((F - 2.27) / 1.82) * (ER - EQ))
1096 R3 = EB + ((W / 2.25) * (EQ - EB))
1098 R4 = EC + ((W / 2.25) * (ER - EC))
1100 R5 = R1 + ((W / 2.25) * (R2 - R1))
1102 R6 = R3 + (((F - 2.27) / 1.82) * (R4 - R3))
1104 R7 = ((R5 + R6) / 2)
1106 T1 = EC + ((Y / 2.25) * (EF - EC))
1108 T2 = ER + ((Y / 2.25) * (EO - ER))
1110 T3 = EC + ((W / 2.25) * (ER - EC))
1112 T4 = EF + ((W / 2.25) * (EO - EF))
1114 T5 = T1 + ((W / 2.25) * (T2 - T1))
1116 T6 = T3 + ((Y / 2.25) * (T4 - T3))
1118 T7 = ((T5 + T6) / 2)
1120 U1 = EB + ((Y / 2.25) * (EE - EB))
1122 U2 = EQ + ((Y / 2.25) * (EN - EQ))
1124 U3 = EB + ((W / 2.25) * (EQ - EB))
1126 U4 = EE + ((W / 2.25) * (EN - EE))
1128 U5 = U1 + ((W / 2.25) * (U2 - U1))
1130 U6 = U3 + ((Y / 2.25) * (U4 - U3))
1132 U7 = ((U5 + U6) / 2)
1134 W1 = A7 + ((W / 2.25) * (P7 - A7))
1136 W2 = R7 + ((Y / 2.25) * (N7 - R7))
1138 W3 = U7 + (((F - 2.27) / 1.82) * (T7 - U7))
1140 W4 = ((W1 + W2 + W3) / 3)
1142 AA = XB + (((F - 2.27) / 1.82) * (XC - XB))
1144 AB = XE + (((F - 2.27) / 1.82) * (XF - XE))
1146 AC = XB + ((Y / 2.25) * (XE - XB))
1148 AD = XC + ((Y / 2.25) * (XF - XC))
1150 AE = AA + ((Y / 2.25) * (AB - AA))
1152 AF = AC + (((F - 2.27) / 1.82) * (AD - AC))
1154 AG = ((AE + AF) / 2)
1156 NA = XE + (((F - 2.27) / 1.82) * (XF - XE))
1158 NB = XN + (((F - 2.27) / 1.82) * (XO - XN))
1160 NC = XE + ((W / 2.25) * (XN - XE))
1162 ND = XF + ((W / 2.25) * (XO - XF))
1164 NE = NA + ((W / 2.25) * (NB - NA))
1166 NF = NC + (((F - 2.27) / 1.82) * (ND - NC))
1168 NG = ((NE + NF) / 2)
1170 PA = XQ + (((F - 2.27) / 1.82) * (XR - XQ))
1172 PB = XN + (((F - 2.27) / 1.82) * (XO - XN))
1174 PC = XQ + ((Y / 2.25) * (XN - XQ))
1176 PD = XR + ((Y / 2.25) * (XO - XR))
1178 PE = PA + ((Y / 2.25) * (PB - PA))
1180 PF = PC + (((F - 2.27) / 1.82) * (PD - PC))
1182 PG = ((PE + PF) / 2)
1184 RA = XB + (((F - 2.27) / 1.82) * (XC - XB))
1186 RB = XQ + (((F - 2.27) / 1.82) * (XR - XQ))
1188 RC = XB + ((W / 2.25) * (XQ - XB))
```

```
1190 RD = XC + ((W / 2.25) * (XR - XC))
1192 RE = RA + ((W / 2.25) * (RB - RA))
1194 RF = RC + (((F - 2.27) / 1.82) * (RD - RC))
1196 RG = ((RE + RF) / 2)
1198 TA = XC + ((Y / 2.25) * (XF - XC))
1200 TB = XR + ((Y / 2.25) * (XO - XR))
1202 TC = XC + ((W / 2.25) * (XR - XC))
1204 TD = XF + ((W / 2.25) * (XO - XF))
1206 TE = TA + ((W / 2.25) * (TB - TA))
1208 TF = TC + ((Y / 2.25) * (TD - TC))
1210 TG = ((TE + TF) / 2)
1212 UA = XB + ((Y / 2.25) * (XE - XB))
1214 UB = XQ + ((Y / 2.25) * (XN - XQ))
1216 UC = XB + ((W / 2.25) * (XQ - XB))
1218 UD = XE + ((W / 2.25) * (XN - XE))
1220 UE = UA + ((W / 2.25) * (UB - UA))
1222 UF = UC + ((Y / 2.25) * (UD - UC))
1224 UG = ((UE + UF) / 2)
1226 WA = AG + ((W / 2.25) * (PG - AG))
1228 WB = RG + ((Y / 2.25) * (NG - RG))
1230 WC = UG + (((F - 2.27) / 1.82) * (TG - UG))
1232 WD = ((WA + WB + WC) / 3)
1234 AH = YB + (((F - 2.27) / 1.82) * (YC - YB))
1236 AI = YE + (((F - 2.27) / 1.82) * (YF - YE))
1238 AJ = YB + ((Y / 2.25) * (YE - YB))
1240 AK = YC + ((Y / 2.25) * (YF - YC))
1242 AL = AH + ((Y / 2.25) * (AI - AH))
1244 AM = AJ + (((F - 2.27) / 1.82) * (AK - AJ))
1246 AN = ((AL + AM) / 2)
1248 NH = YE + (((F - 2.27) / 1.82) * (YF - YE))
1250 NI = YN + (((F - 2.27) / 1.82) * (YO - YN))
1252 NJ = YE + ((W / 2.25) * (YN - YE))
1254 NK = YF + ((W / 2.25) * (YO - YF))
1256 NL = NH + ((W / 2.25) * (NI - NH))
1258 NM = NJ + (((F - 2.27) / 1.82) * (NK - NJ))
1260 NN = ((NL + NM) / 2)
1262 PH = YQ + (((F - 2.27) / 1.82) * (YR - YQ))
1264 PI = YN + (((F - 2.27) / 1.82) * (YO - YN))
1266 PJ = YQ + ((Y / 2.25) * (YN - YQ))
1268 PK = YR + ((Y / 2.25) * (YO - YR))
1270 PL = PH + ((Y / 2.25) * (PI - PH))
1272 PM = PJ + (((F - 2.27) / 1.82) * (PK - PJ))
1274 PN = ((PL + PM) / 2)
1276 RH = YB + (((F - 2.27) / 1.82) * (YC - YB))
1278 RI = YQ + (((F - 2.27) / 1.82) * (YR - YQ))
1280 RJ = YB + ((W / 2.25) * (YQ - YB))
1282 RK = YC + ((W / 2.25) * (YR - YC))
1284 RL = RH + ((W / 2.25) * (RI - RH))
1286 RM = RJ + (((F - 2.27) / 1.82) * (RK - RJ))
1288 RN = ((RL + RM) / 2)
1290 TH = YC + ((Y / 2.25) * (YF - YC))
1292 TI = YR + ((Y / 2.25) * (YO - YR))
1294 TJ = YC + ((W / 2.25) * (YR - YC))
1296 TK = YF + ((W / 2.25) * (YO - YF))
1298 TL = TH + ((W / 2.25) * (TI - TH))
1300 TM = TJ + ((Y / 2.25) * (TK - TJ))
1302 TN = ((TL + TM) / 2)
1304 UH = YB + ((Y / 2.25) * (YE - YB))
1306 UI = YQ + ((Y / 2.25) * (YN - YQ))
1308 UJ = YB + ((W / 2.25) * (YQ - YB))
1310 UK = YE + ((W / 2.25) * (YN - YE))
1312 UL = UH + ((W / 2.25) * (UI - UH))
1314 UM = UJ + ((Y / 2.25) * (UK - UJ))
1316 UN = ((UL + UM) / 2)
1318 WI = AN + ((W / 2.25) * (PN - AN))
1320 WJ = RN + ((Y / 2.25) * (NN - RN))
```

```
1322 WK = UN + (((F - 2.27) / 1.82) * (TN - UN))
1324 WL = ((WI + WJ + WK) / 3)
1326 AO = ZB + (((F - 2.27) / 1.82) * (ZC - ZB))
1328 AP = ZE + (((F - 2.27) / 1.82) * (ZF - ZE))
1330 AQ = ZB + ((Y / 2.25) * (ZE - ZB))
1332 AR = ZC + ((Y / 2.25) * (ZF - ZC))
1334 AS = AO + ((Y / 2.25) * (AP - AO))
1336 AX = AQ + (((F - 2.27) / 1.82) * (AR - AQ))
1338 AU = ((AS + AX) / 2)
1340 NO = ZE + (((F - 2.27) / 1.82) * (ZF - ZE))
1342 NP = ZN + (((F - 2.27) / 1.82) * (ZO - ZN))
1344 NQ = ZE + ((W / 2.25) * (ZN - ZE))
1346 NR = ZF + ((W / 2.25) * (ZO - ZF))
1348 NS = NO + ((W / 2.25) * (NP - NO))
1350 NT = NQ + (((F - 2.27) / 1.82) * (NR - NQ))
1352 NU = ((NS + NT) / 2)
1354 PO = ZQ + (((F - 2.27) / 1.82) * (ZR - ZQ))
1356 PP = ZN + (((F - 2.27) / 1.82) * (ZO - ZN))
1358 PQ = ZQ + ((Y / 2.25) * (ZN - ZQ))
1360 PR = ZR + ((Y / 2.25) * (ZO - ZR))
1362 PS = PO + ((Y / 2.25) * (PP - PO))
1364 PT = PQ + (((F - 2.27) / 1.82) * (PR - PQ))
1366 PU = ((PS + PT) / 2)
1368 RO = ZB + (((F - 2.27) / 1.82) * (ZC - ZB))
1370 RP = ZQ + (((F - 2.27) / 1.82) * (ZR - ZQ))
1372 RQ = ZB + ((W / 2.25) * (ZQ - ZB))
1374 RR = ZC + ((W / 2.25) * (ZR - ZC))
1376 RS = RO + ((W / 2.25) * (RP - RO))
1378 RT = RQ + (((F - 2.27) / 1.82) * (RR - RQ))
1380 RU = ((RS + RT) / 2)
1382 TX = ZC + ((Y / 2.25) * (ZF - ZC))
1384 TP = ZR + ((Y / 2.25) * (ZO - ZR))
1386 TQ = ZC + ((W / 2.25) * (ZR - ZC))
1388 TR = ZF + ((W / 2.25) * (ZO - ZF))
1390 TS = TX + ((W / 2.25) * (TP - TX))
1392 TT = TQ + ((Y / 2.25) * (TR - TQ))
1394 TU = ((TS + TT) / 2)
1396 UO = ZB + ((Y / 2.25) * (ZE - ZB))
1398 UP = ZQ + ((Y / 2.25) * (ZN - ZQ))
1400 UQ = ZB + ((W / 2.25) * (ZQ - ZB))
1402 UR = ZE + ((W / 2.25) * (ZN - ZE))
1404 US = UO + ((W / 2.25) * (UP - UO))
1406 UT = UQ + ((Y / 2.25) * (UR - UQ))
1408 UU = ((US + UT) / 2)
1410 WQ = AU + ((W / 2.25) * (PU - AU))
1412 WR = RU + ((Y / 2.25) * (NU - RU))
1414 WS = UU + (((F - 2.27) / 1.82) * (TU - UU))
1416 WT = ((WQ + WR + WS) / 3)
1420  GOSUB 1960
1422  INPUT "DO YOU WANT A HARDCOPY? ENTER Y(YES) OR N(NO) ";A$
1423  IF A$ = "N" GOTO 4005
1424  IF A$ = "Y" THEN  PR# 1: GOSUB 1965
1425  PR# 0
1426  GOTO 4005
1430 W =  - X
1435  REM  EQUATIONS FOR QUADRANT D
1440 B1 = EA + (((F - 1.45) / .82) * (EB - EA))
1442 B2 = ED + (((F - 1.45) / .82) * (EE - ED))
1444 B3 = EA + ((Y / 2.25) * (ED - EA))
1446 B4 = EB + ((Y / 2.25) * (EE - EB))
1448 B5 = B1 + ((Y / 2.25) * (B2 - B1))
1450 B6 = B3 + (((F - 1.45) / .82) * (B4 - B3))
1452 B7 = ((B5 + B6) / 2)
1454 O1 = ED + (((F - 1.45) / .82) * (EE - EI ))
1456 O2 = EM + (((F - 1.45) / .82) * (EN - E  ))
1458 O3 = ED + ((W / 2.25) * (EM - ED))
```

```
1460 O4 = EE + ((W / 2.25) * (EN - EE))
1462 O5 = O1 + ((W / 2.25) * (O2 - O1))
1464 O6 = O3 + (((F - 1.45) / .82) * (O4 - O3))
1466 O7 = ((O5 + O6) / 2)
1468 Q1 = EP + (((F - 1.45) / .82) * (EQ - EP))
1470 Q2 = EM + (((F - 1.45) / .82) * (EN - EM))
1472 Q3 = EP + ((Y / 2.25) * (EM - EP))
1474 Q4 = EQ + ((Y / 2.25) * (EN - EQ))
1476 Q5 = Q1 + ((Y / 2.25) * (Q2 - Q1))
1478 Q6 = Q3 + (((F - 1.45) / .82) * (Q4 - Q3))
1480 Q7 = ((Q5 + Q6) / 2)
1482 S1 = EA + (((F - 1.45) / .82) * (EB - EA))
1484 S2 = EP + (((F - 1.45) / .82) * (EQ - EP))
1486 S3 = EA + ((W / 2.25) * (EP - EA))
1488 S4 = EB + ((W / 2.25) * (EQ - EB))
1490 S5 = S1 + ((W / 2.25) * (S2 - S1))
1492 S6 = S3 + (((F - 1.45) / .82) * (S4 - S3))
1494 S7 = ((S5 + S6) / 2)
1496 U1 = EB + ((Y / 2.25) * (EE - EB))
1498 U2 = EQ + ((Y / 2.25) * (EN - EQ))
1500 U3 = EB + ((W / 2.25) * (EQ - EB))
1502 U4 = EE + ((W / 2.25) * (EN - EE))
1504 U5 = U1 + ((W / 2.25) * (U2 - U1))
1506 U6 = U3 + ((Y / 2.25) * (U4 - U3))
1508 U7 = ((U5 + U6) / 2)
1510 V1 = EA + ((Y / 2.25) * (ED - EA))
1512 V2 = EP + ((Y / 2.25) * (EM - EP))
1514 V3 = EA + ((W / 2.25) * (EP - EA))
1516 V4 = ED + ((W / 2.25) * (EM - ED))
1518 V5 = V1 + ((W / 2.25) * (V2 - V1))
1520 V6 = V3 + ((Y / 2.25) * (V4 - V3))
1522 V7 = ((V5 + V6) / 2)
1524 W5 = S7 + ((W / 2.25) * (Q7 - S7))
1526 W6 = S7 + ((Y / 2.25) * (O7 - S7))
1528 W7 = V7 + (((F - 1.45) / .82) * (U7 - V7))
1530 W8 = ((W5 + W6 + W7) / 3)
1532 BA = XA + (((F - 1.45) / .82) * (XB - XA))
1534 BB = XD + (((F - 1.45) / .82) * (XE - XD))
1536 BC = XA + ((Y / 2.25) * (XD - XA))
1538 BD = XB + ((Y / 2.25) * (XE - XB))
1540 BE = BA + ((Y / 2.25) * (BB - BA))
1542 BF = BC + (((F - 1.45) / .82) * (BD - BC))
1544 BG = ((BE + BF) / 2)
1546 OA = XD + (((F - 1.45) / .82) * (XE - XD))
1548 OB = XM + (((F - 1.45) / .82) * (XN - XM))
1550 OC = XD + ((W / 2.25) * (XM - XD))
1552 OD = XE + ((W / 2.25) * (XN - XE))
1554 OE = OA + ((W / 2.25) * (OB - OA))
1556 OF = OC + (((F - 1.45) / .82) * (OD - OC))
1558 OG = ((OE + OF) / 2)
1560 QA = XF + (((F - 1.45) / .82) * (XG - XF))
1562 QB = XM + (((F - 1.45) / .82) * (XN - XM))
1564 QC = XF + ((Y / 2.25) * (XM - XF))
1566 QD = XG + ((Y / 2.25) * (XN - XG))
1568 QE = QA + ((Y / 2.25) * (QB - QA))
1570 QF = QC + (((F - 1.45) / .82) * (QD - QC))
1572 QG = ((QE + QF) / 2)
1574 SA = XA + (((F - 1.45) / .82) * (XB - XA))
1576 SB = XF + (((F - 1.45) / .82) * (XG - XF))
1578 SC = XA + ((W / 2.25) * (XF - XA))
1580 SD = XB + ((W / 2.25) * (XG - XB))
1582 SE = SA + ((W / 2.25) * (SB - SA))
1584 SF = SC + (((F - 1.45) / .82) * ([illegible] - SC))
1586 SG = (([illegible] + SF) / 2)
1588 [illegible] = [illegible] / 2.25) * [illegible]
1590 [illegible] = [illegible] / 2.25) * [illegible]
```

```
1592 UC = XB + ((W / 2.25) * (XQ - XB))
1594 UD = XE + ((W / 2.25) * (XN - XE))
1596 UE = UA + ((W / 2.25) * (UB - UA))
1598 UF = UC + ((Y / 2.25) * (UD - UC))
1600 UG = ((UE + UF) / 2)
1602 VA = XA + ((Y / 2.25) * (XD - XA))
1604 VB = XP + ((Y / 2.25) * (XM - XP))
1606 VC = XA + ((W / 2.25) * (XP - XA))
1608 VD = XD + ((W / 2.25) * (XM - XD))
1610 VE = VA + ((W / 2.25) * (VB - VA))
1612 VF = VC + ((Y / 2.25) * (VD - VC))
1614 VG = ((VE + VF) / 2)
1616 WE = BG + ((W / 2.25) * (QG - BG))
1618 WF = SG + ((Y / 2.25) * (OG - SG))
1620 WG = VG + (((F - 1.45) / .82) * (UG - VG))
1622 WH = ((WE + WF + WG) / 3)
1624 BH = YA + (((F - 1.45) / .82) * (YB - YA))
1626 BI = YD + (((F - 1.45) / .82) * (YE - YD))
1628 BJ = YA + ((Y / 2.25) * (YD - YA))
1630 BK = YB + ((Y / 2.25) * (YE - YB))
1632 BL = BH + ((Y / 2.25) * (BI - BH))
1634 BM = BJ + (((F - 1.45) / .82) * (BK - BJ))
1636 BN = ((BL + BM) / 2)
1638 OH = YD + (((F - 1.45) / .82) * (YE - YD))
1640 OI = YM + (((F - 1.45) / .82) * (YN - YM))
1642 OJ = YD + ((W / 2.25) * (YM - YD))
1644 OK = YE + ((W / 2.25) * (YN - YE))
1646 OL = OH + ((W / 2.25) * (OI - OH))
1648 OM = OJ + (((F - 1.45) / .82) * (OK - OJ))
1650 OX = ((OL + OM) / 2)
1652 QH = YP + (((F - 1.45) / .82) * (YQ - YP))
1654 QI = YM + (((F - 1.45) / .82) * (YN - YM))
1656 QJ = YP + ((Y / 2.25) * (YM - YP))
1658 QK = YQ + ((Y / 2.25) * (YN - YQ))
1660 QL = QH + ((Y / 2.25) * (QI - QH))
1662 QM = QJ + (((F - 1.45) / .82) * (QK - QJ))
1664 QN = ((QL + QM) / 2)
1666 SH = YA + (((F - 1.45) / .82) * (YB - YA))
1668 SI = YP + (((F - 1.45) / .82) * (YQ - YP))
1670 SJ = YA + ((W / 2.25) * (YP - YA))
1672 SK = YB + ((W / 2.25) * (YQ - YB))
1674 SL = SH + ((W / 2.25) * (SI - SH))
1676 SM = SJ + (((F - 1.45) / .82) * (SK - SJ))
1678 SN = ((SL + SM) / 2)
1680 UH = YB + ((Y / 2.25) * (YE - YB))
1682 UI = YQ + ((Y / 2.25) * (YN - YQ))
1684 UJ = YB + ((W / 2.25) * (YQ - YB))
1686 UK = YE + ((W / 2.25) * (YN - YE))
1688 UL = UH + ((W / 2.25) * (UI - UH))
1690 UM = UJ + ((Y / 2.25) * (UK - UJ))
1692 UN = ((UL + UM) / 2)
1694 VH = YA + ((Y / 2.25) * (YD - YA))
1696 VI = YP + ((Y / 2.13) * (YM - YP))
1698 VJ = YA + ((W / 2.25) * (YP - YA))
1700 VK = YD + ((W / 2.25) * (YM - YD))
1702 VL = VH + ((W / 2.25) * (VI - VH))
1704 VM = VJ + ((Y / 2.25) * (VK - VJ))
1706 VN = ((VL + VM) / 2)
1708 WM = BN + ((W / 2.25) * (ON - BN))
1710 WN = SN + ((Y / 2.25) * (QN - SN))
1712 WO = VN + (((F - 1.45) / .82) * (UN - VN))
1714 WP = ((WM + WN + WO) / 3)
1716 BO = ZA + (((F - 1.45) / .82) * (ZB - ZA))
1718 BP = ZD + (((F - 1.45) / .82) * (ZE - ZD))
1720 BQ = ZA + ((Y / 2.25) * (ZD - ZA))
1722 BR = ZB + ((Y / 2.25) * (ZE - ZB))
```

```
1724 BS = BO + ((Y / 2.25) * (BP - BO))
1726 BT = BQ + (((F - 1.45) / .82) * (BR - BQ))
1728 BU = ((BS + BT) / 2)
1730 OO = ZD + (((F - 1.45) / .82) * (ZE - ZD))
1732 OP = ZM + (((F - 1.45) / .82) * (ZN - ZM))
1734 OQ = ZD + ((W / 2.25) * (ZM - ZD))
1736 OY = ZE + ((W / 2.25) * (ZN - ZE))
1738 OS = OO + ((W / 2.25) * (OP - OO))
1740 OT = OQ + (((F - 1.45) / .82) * (OY - OQ))
1742 OU = ((OS + OT) / 2)
1744 QO = ZP + (((F - 1.45) / .82) * (ZQ - ZP))
1746 QP = ZM + (((F - 1.45) / .82) * (ZN - ZM))
1748 QQ = ZP + ((Y / 2.25) * (ZM - ZP))
1750 QR = ZQ + ((Y / 2.25) * (ZN - ZQ))
1752 QS = QO + ((Y / 2.25) * (QP - QO))
1754 QT = QQ + (((F - 1.45) / .82) * (QR - QQ))
1756 QU = ((QS + QT) / 2)
1758 SO = ZA + (((F - 1.45) / .82) * (ZB - ZA))
1760 SP = ZP + (((F - 1.45) / .82) * (ZQ - ZP))
1762 SQ = ZA + ((W / 2.25) * (ZP - ZA))
1764 SR = ZB + ((W / 2.25) * (ZQ - ZB))
1766 SS = SO + ((W / 2.25) * (SP - SO))
1768 ST = SQ + (((F - 1.45) / .82) * (SR - SQ))
1770 SU = ((SS + ST) / 2)
1772 UO = ZB + ((Y / 2.25) * (ZE - ZB))
1774 UP = ZQ + ((Y / 2.25) * (ZN - ZQ))
1776 UQ = ZB + ((W / 2.25) * (ZQ - ZB))
1778 UR = ZE + ((W / 2.25) * (ZN - ZE))
1780 US = UO + ((W / 2.25) * (UP - UO))
1782 UT = UQ + ((Y / 2.25) * (UR - UQ))
1784 UU = ((US + UT) / 2)
1786 VO = ZA + ((Y / 2.25) * (ZD - ZA))
1788 VP = ZP + ((Y / 2.25) * (ZM - ZP))
1790 VQ = ZA + ((W / 2.25) * (ZP - ZA))
1792 VR = ZD + ((W / 2.25) * (ZM - ZD))
1794 VS = VO + ((W / 2.25) * (VP - VO))
1796 VT = VQ + ((Y / 2.25) * (VR - VQ))
1798 VU = ((VS + VT) / 2)
1800 WU = BU + ((W / 2.25) * (QU - BU))
1802 WV = SU + ((Y / 2.25) * (OU - SU))
1804 WW = VU + (((F - 1.45) / .82) * (UU - VU))
1806 WX = ((WU + WV + WW) / 3)
1810  GOSUB 2025
1812  INPUT "DO YOU WANT A HARDCOPY? ENTER Y(YES) OR N(NO) ";A$
1813  IF A$ = "N" GOTO 4005
1814  IF A$ = "Y" THEN  PR# 1: GOSUB 2030
1815  PR# 0
1816  GOTO 4005
1830  REM   RESULTS FOR QUADRANT A
1835  HOME
1840  PRINT "RESULTS FOR THE FOLLOWING INPUTS ARE": PRINT
1845  PRINT "LOAD = ";F: PRINT
1850  PRINT "X-AXIS = ";X: PRINT
1855  PRINT "Y-AXIS = ";YT: PRINT
1859  PRINT "  ** RESULTS **": PRINT
1860  PRINT "ENDURANCE FORWARD = ";M4: PRINT
1865  PRINT "STD.DEVIATION FORWARD = ";ML: PRINT
1870  PRINT "ENDURANCE LATERAL = ";MD: PRINT
1875  PRINT "STD.DEVIATION LATERAL = ";MT: PRINT
1880  RETURN
1890  REM   RESULTS FOR QUADRANT B
1895  HOME
1900  PRINT "THE RESULTS FOR THE FOLLOWING INPUTS ARE": PRINT
1905  PRINT "LOAD = ";F: PRINT
1910  PRINT "X-AXIS = ";X: PRINT
1915  PRINT "Y-AXIS = ";YT: PRINT
```

```
1920  PRINT "  ** RESULTS **": PRINT
1925  PRINT "ENDURANCE FORWARD = ";[illegible]: PRINT
1930  PRINT "STD. DEVIATION FORWARD = ";MP: PRINT
1935  PRINT "ENDURANCE LATERAL = ";MH: PRINT
1940  PRINT "STD. DEVIATION LATERAL = ";MX: PRINT
1945  RETURN
1955  REM  RESULTS FOR QUADRANT C
1960  HOME
1965  PRINT "THE RESULTS FOR THE FOLLOWING INPUTS ARE": PRINT
1970  PRINT "LOAD = "F: PRINT
1975  PRINT "X-AXIS = ";X: PRINT
1980  PRINT "Y-AXIS = ";YT: PRINT
1985  PRINT "  ** RESULTS **": PRINT
1990  PRINT "ENDURANCE FORWARD = ";W4: PRINT
1995  PRINT "STD.DEVIATION FORWARD = ";WL: PRINT
2000  PRINT "ENDURANCE LATERAL = ";WD: PRINT
2005  PRINT "STD.DEVIATION LATERAL = ";WT: PRINT
2010  RETURN
2020  REM   RESULTS FOR QUADRANT D
2025  HOME
2030  PRINT "THE RESULTS FOR THE FOLLOWING INPUTS ARE": PRINT
2035  PRINT "LOAD = ";F: PRINT
2040  PRINT "X-AXIS = ";X: PRINT
2045  PRINT "Y-AXIS = ";YT: PRINT
2050  PRINT "  ** RESULTS **": PRINT
2055  PRINT "ENDURANCE FORWARD = ";W8: PRINT
2060  PRINT "STD.DEVIATION FORWARD = ";WF: PRINT
2065  PRINT "ENDURANCE LATERAL = ";WH: PRINT
2070  PRINT "STD.DEVIATION LATERAL = ";WX: PRINT
2075  RETURN
4000  PRINT "DATA OUT OF BOUNDS": PRINT
4005  PRINT : PRINT "TO REENTER DATA"
4006  PRINT "OR TO ENTER NEW DATA TYPE 1 ": PRINT
4010  INPUT "TO END PROGRAM TYPE 0 ";Z
4015  IF Z = 1 GOTO 159
4020  IF Z = 0 GOTO 9999
9999  END
```

APPENDIX D

COMPUTER LISTING (THE TEST PROGRAM)

```
5  REM : ARMY WELCH PROGRAM
10  PRINT "ONE-TAILED WELCH TEST PROGRAM"
20  PRINT "NOTE: THIS PROGRAM TESTS FOR WHETHER A LARGER (SMALLER) ENDURAN
     CE TIME FOR ONE HEADGEAR CONFIGURATION"
25  PRINT "IS SIGNIFICANTLY LARGER (SMALLER) COMPARED TO A SECOND HEADGEAR
     CONFIGURATION."
30  REM : C.A.P. 27-MAY-84.
50  PRINT
105  INPUT "WHAT IS THE MEAN VALUE OF THE FORWARD ENDURANCE TIME FOR THE F
     IRST HEADGEAR CONFIGURATION ?";A1
107  PRINT
110  INPUT "WHAT IS THE STANDARD DEVIATION OF THE FORWARD ENDURANCE TIME F
     OR THE FIRST HEADGEAR CONFIGURATION ?";Q1
115 V1 = Q1 * Q1
122  PRINT
125  INPUT "WHAT IS THE MEAN VALUE OF THE LATERAL ENDURANCE TIME FOR THE F
     IRST HEADGEAR CONFIGURATION ?";A3
128  PRINT
130  INPUT "WHAT IS THE STANDARD DEVIATION OF THE LATERAL ENDURANCE TIME F
     OR THE FIRST HEAD GEAR CONFIGURATION ?";Q3
135 V3 = Q3 * Q3
140  PRINT
145  INPUT "WHAT IS THE MEAN VALUE OF THE FORWARD ENDURANCE TIME FOR THE S
     ECOND HEAD GEAR CONFIGURATION ?";A2
148  PRINT
150  INPUT "WHAT IS THE STANDARD DEVIATION OF THE FORWARD ENDURANCE TIME F
     OR THE SECOND HEADGEAR CONFIGURATION ?";Q2
155 V2 = Q2 * Q2
160  PRINT
165  INPUT "WHAT IS THE MEAN VALUE OF THE LATERAL ENDURANCE TIME FOR THE S
     ECOND HELMET CONFIGURATION ?";A4
168  PRINT
170  INPUT "WHAT IS THE STANDARD DEVIATION OF THE LATERAL ENDURANCE TIME F
     OR THE SECOND HEADGEAR CONFIGURATION ?";Q4
175 V4 = Q4 * Q4
300 Z1 = V1 / 6:Z2 = V2 / 6
305 Z3 = V3 / 6:Z4 = V4 / 6
310 W1 = (A1 - A2) /  SQR (Z1 + Z2)
315 W2 =  ABS (W1)
317 W3 = (A3 - A4) /  SQR (Z3 + Z4)
319 W4 =  ABS (W3)
350  PR# 1
400  PRINT
450  PRINT
500  PRINT "FOR THE FOLLOWING CONDITIONS:"
502  PRINT
505  PRINT "HEADGEAR CONFIGURATION #1:"
510  PRINT "FORWARD ENDURANCE TIME: ";A1;" SECONDS."
```

```
512  PRINT "(STD. DEV.= ";Q1;" SECS.)"
515  PRINT "HEADGEAR CONFIGURATION #2:"
520  PRINT "FORWARD ENDURANCE TIME: ";A2;" SEDONDS."
522  PRINT "(STD. DEV.= ";Q2;" SECS.)"
525  PRINT
530  PRINT "THE CALCULATED WELCH NUMBER: ";W2
550 X1 = W2 - 2.01
555  IF X1 < 0 THEN  GOTO 600
560 X2 = W2 - 2.57
565  IF X2 < 0 THEN  GOTO 610
570 X3 = W2 - 4.03
575  IF X3 < 0 THEN  GOTO 620
580  GOTO 630
600  PRINT "THIS IS NOT SIGNIFICANT (p>.05)."
605  GOTO 700
610  PRINT "THIS IS SIGNIFICANT AT THE 95% CONFIDENCE LEVEL (p<.05)."
615  GOTO 700
620  PRINT "THIS IS SIGNIFICANT AT THE 97.5% CONFIDENCE LEVEL (p<.025)."
625  GOTO 700
630  PRINT "THIS IS SIGNIFICANT AT THE 99.5% CONFIDENCE LEVEL (p<,005)."
635  GOTO 700
700  PRINT
702  PRINT
704  PRINT "FOR THE FOLLOWING CONDITIONS:"
706  PRINT
708  PRINT "HEDGEAR CONFIGURATION #1:"
710  PRINT "LATERAL ENDURANCE TIME: ";A3;" SECONDS."
712  PRINT "(STD. DEV.= ";Q3;" SECS.)"
715  PRINT "HEADGEAR CONFIGURATION #2:"
720  PRINT "LATERAL ENDURANCE TIME: ";A4;" SECONDS."
722  PRINT "(STD. DEV.= ";Q4;" SECS.)"
725  PRINT
727  PRINT "THE CALCULATED WELCH NUMBER: ";W4
750 X4 = W4 - 2.01
755  IF X4 < 0 THEN  GOTO 800
760 X5 = W4 - 2.57
765  IF X5 < 0 THEN  GOTO 810
770 X6 = W4 - 4.03
775  IF X6 < 0 THEN  GOTO 820
780  GOTO 830
800  PRINT "THIS IS NOT SIGNIFICANT (p>.05)"
805  GOTO 890
810  PRINT "THIS IS SIGNIFICANT AT THE 95% CONFIDENCE LEVEL (p<.05)."
815  GOTO 890
820  PRINT "THIS IS SIGNIFICANT AT THE 97.5% CONFIDENCE LEVEL (p<.025)."
825  GOTO 890
830  PRINT "THIS IS SIGNIFICANT AT THE 99.5% CONFIDENCE LEVEL (p<.005)."
835  GOTO 890
890  PRINT
895  PRINT
899  PR# 0
900  INPUT "DO YOU WISH TO EVALUATE ANOTHER PAIR OF HEADGEAR CONFIGURATION
     S? TYPE '1' FOR 'YES' OR TYPE '2' FOR 'NO'.";P1
910  IF P1 = 1 THEN  GOTO 50
999  END
```

END

7-87

DTIC